1.12 **a.** $\dfrac{1220\text{ m} - 1000\text{ m}}{4\text{ hr}} = 55\text{ m/hr}$

b. $\dfrac{655{,}000 - 534{,}000}{1990 - 1980} = \dfrac{121{,}000\text{ people}}{10\text{ yr}}$

$= 12{,}100\text{ people/yr}$

c. $\dfrac{\$32 - \$8}{1990 - 1950} = \dfrac{\$24}{40\text{ yr}} = \$0.60\text{/yr}$

d. $\dfrac{1500 - 2400}{3 - 1} = \dfrac{-900\text{ m}}{2\text{ sec}} = -450\text{ m/sec}$

e. Sample response: The domain is the times from 1 sec to 3 sec. The range is the altitudes from 2400 m to 1500 m.

ACTIVITY 2

This activity introduces students to the slope-intercept form of linear equations.

Materials List

- none

Technology

- spreadsheet
- graphing utility
- symbolic manipulator (optional)

teacher note

Table **3-5** gives only a sample listing of breakfast foods. You can obtain complete lists from the original sources.

Student Outcomes

After completing the following exploration and discussion, students should be able to:

- ✶ interpret data from a table
- ✶ use a graphing utility to display data
- ✶ describe slope as a rate
- ✶ describe the slopes of parallel lines
- ✶ write a linear equation given the slope and the y-intercept.

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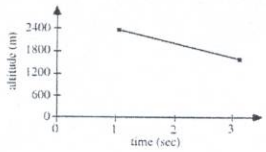
1.12 The slope of a line can be used to describe the average rate of change in one quantity with respect to another. For example, one familiar rate of change is speed, often expressed in kilometers per hour.

a. Rolando is climbing a mountain. At 10:00 A.M., he stood at an elevation of 1000 m. By 2:00 P.M., he had reached an elevation of 1220 m. Determine Rolando's average rate of change in elevation in meters per hour.

b. In 1980, the city of Tucson, Arizona, had a population of 534,000. By 1990, its population had grown to 655,000. Find Tucson's average rate of change in population in people per year.

c. In 1950, the price of a pair of jeans was about $8.00. By 1990, the price had risen to about $32.00. Find the average rate of change in the price of jeans in dollars per year.

d. The graph below shows the change in a parachutist's altitude (in meters) during an interval of 2 sec. Use the graph to estimate the parachutist's average rate of change in altitude in meters per second.



e. Identify the domain and range for the graph in Part d.

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ACTIVITY 2

Many dietary specialists think of breakfast as the most important meal of the day. In this activity, you use linear equations to help plan an adequate breakfast for an active morning. Table 3-5 shows the number of kilocalories per serving in some typical breakfast foods.

TABLE 3-5 ■ *Kilocalories Per Serving for Common Breakfast Foods*

Food	kcal/item	Food	kcal/item
Toast, white	80	Croissant, egg, bacon, cheese	386
Toast, wheat	70	Biscuit, bacon, egg, cheese	483
Doughnut, plain	240	Biscuit with sausage	330
Cereal, with sugar	180	Cherry pie	260
Cereal, plain	120	Egg with muffin	340
Apple	60	English muffin with butter	186
Banana	80	Hotcakes with butter	500
Grapefruit	60	French toast	400
Orange juice	120	Fries	360
Egg, fried	120	Omelet	290
Egg, scrambled	80	Sausage, one patty	200
Egg, substitute	90	Milk, 2%	112
Milk, whole	160	Milk, chocolate	192
Yogurt, plain	120	Peanut butter and jam	500
Coffee	0	Soda pop	144

SOURCES: Gebhardt and Matthews, *Nutritive Value of Foods*; McArdle, et al., *Exercise Physiology*; Page and Raper, *Food and Your Weight*.

Exploration

a. Use the information in Tables 3-3 and 3-5 to design a breakfast that will supply a 62-kg person with the number of kilocalories necessary to play racquetball for 1 hr.

b. Make a table that shows the kilocalories remaining from the meal at the end of each 5-min interval of racquetball.

c. Create a scatterplot of the data from Part b. Let the time in minutes be the domain of the relation and the energy remaining be the range.

d. Determine the slope of the data.

e. Determine the coordinates of the point where the data intersects the y-axis.

mathematics note

The y-coordinate of the point where a line intersects the y-axis is known as the **y-intercept.**

The equation of a line with slope m and y-intercept b can be written in the form $y = mx + b$. This is the **slope-intercept** form of the equation of a line.

For example, Figure 3-3 shows a graph of the equation $y = -2x + 4$. This line has a slope of -2 and a y-intercept of 4. It intersects the y-axis at the point (0,4).

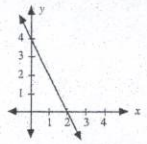


FIGURE 3-3 Graph of $y = -2x + 4$.

f. On the same set of axes as the scatterplot in Part c, graph the equation $y = mx + b$, where m is the slope of data and b is the y-intercept.

g. Repeat Parts b–f for a 62-kg person who ate a breakfast of 800 kcal.

Discussion

a. In Part f of the exploration, how did the scatterplot compare with the graph of the equation in the form $y = mx + b$?

b. Describe what the variables y and x represent in the exploration.

c. What information besides the slope do you need to identify a specific line?

d. Compare the slopes of the equations in Parts f and g of the exploration.

Exploration

a. The kilocalories needed for a 62-kg person to play racquetball for 1 hr can be found as follows:

$$\left(0.178 \frac{\text{kcal}}{\text{min} \bullet \text{kg}}\right)(60 \text{ min})(62 \text{ kg}) \approx 662 \text{ kcal}$$

An adequate breakfast might include a glass of 2% milk (112 kcal), an omelet (290 kcal), a glass of orange juice (120 kcal), and two pieces of wheat toast (140 kcal).

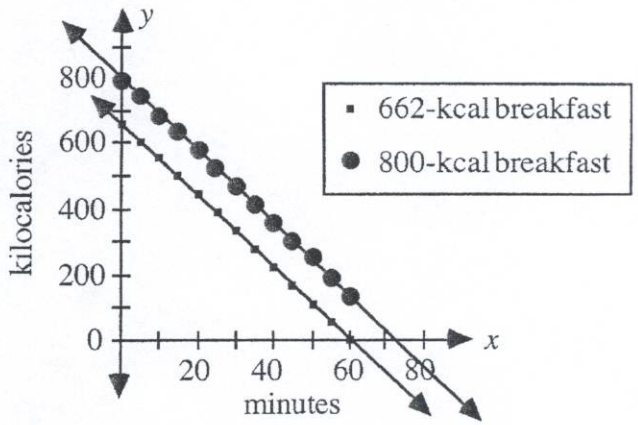
b–g. While playing racquetball, a 62-kg person burns approximately 55 kcal every 5 min. See the sample table below.

Both graphs have the same slope. Using the points (5,607) and (10,552), for example, the slope is:

$$\frac{607 - 552}{5 - 10} = -11$$

The graphs intersect the y-axis at 662 and 800, respectively.

The equation that models the energy remaining after a 662-kcal breakfast is $y = -11x + 662$. The equation that models the energy remaining after a 800-kcal breakfast is $y = -11x + 800$. The following graph shows both equations along with the respective scatterplots.



Discussion

a. Sample response: The graph of the equation goes through the points on the scatterplot.

b. In this situation, y represents the kilocalories remaining from breakfast as the person plays racquetball, while x represents the minutes that the person plays racquetball.

c. The two identifying characteristics of a line are slope and y-intercept.

d. The slopes of the equations are equal; therefore, the graphs of the equations are parallel.

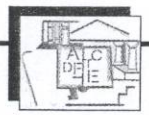
Time (min)	Energy Remaining from 662-kcal Meal	Energy Remaining from 800-kcal Meal
0	662	800
5	607	745
10	552	690
⋮	⋮	⋮
55	57	195
60	2	140

e. When a linear equation is written in slope-intercept form, the slope is the coefficient of x, while the y-intercept is the constant. In the equation $y = 7x + 6$, for example, the slope is 7 and the y-intercept is 6.

f. Sample response: Given the slope m and the y-intercept b of a line, the general form of its equation is $y = mx + b$.

Warm-Up

1. **a.** $m = 3$, $b = 1$ **b.** $m = -2$, $b = 5$
   c. $m = 2/3$, $b = -5$ **d.** $m = -1$, $b = -3$
2. **a.** $y = 7x - 3$ **b.** $y = (1/4)x - 5$
   c. $y = -(2/5)x + 7$
3. **a.** Both lines have a slope of 3, so they are parallel.
   b. The two lines have different slopes, 1/4 and 3/4, so they are not parallel.
   c. Both lines have a slope of −2, so they are parallel.



teacher note

Students might find symbolic manipulators helpful in the assignment. In Problem **2.7**, students write an equation of a line in point-slope form.

Assignment

Problems suitable for use as assessment items are identified by an asterisk (*).

* 2.1 **a.** The kilocalories needed for a 50-kg person to dance aerobically for 1 hr can be found as follows:

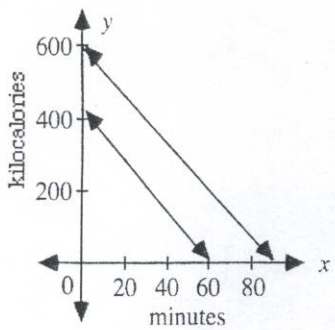
$$\left(0.135 \frac{\text{kcal}}{\text{min} \bullet \text{kg}}\right)(60 \text{ min})(50 \text{ kg}) = 405 \text{ kcal}$$

One possible breakfast includes wheat toast (70 kcal) and an egg with muffin (340 kcal) for a total of 410 kcal.

b. The corresponding equation for the sample breakfast in Part **a** is $y = -6.75x + 410$, where y represents the kilocalories remaining from the meal and x represents time in minutes.

c. $y = -6.75x + 600$

d. Sample graph:



mathematics note

Two lines that have equal slopes are **parallel**.

As shown in Figure 3-4, for example, the graphs of the lines $y = 2x$ and $y = 2x + 3$ are parallel.

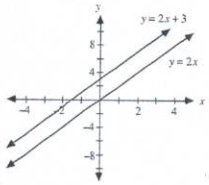


Figure 3-4 Graph of $y = 2x$ and $y = 2x + 3$.

Although vertical lines have no slope, any two vertical lines are also parallel.

e. Given the equation of a line in the form $y = mx + b$, describe how to determine its slope and y-intercept.

f. Given the slope and y-intercept of a line, describe how to write an equation of the line.

Warm-Up

1. Identify the slope and y-intercept of each of the following lines.
   a. $y = 3x + 1$
   b. $y = -2x + 5$
   c. $y = (2/3)x - 5$
   d. $y = -1x - 3$
2. Write an equation in slope-intercept form for each of the following:
   a. the line with a slope of 7 and y-intercept of −3
   b. the line that crosses the y-axis at (0,−5) and has a slope of 1/4
   c. the line with a slope of −2/5 and y-intercept of 7

3. Identify each of the following pairs of lines as parallel or not parallel. Justify your responses.
   a. $y = 3x - 5$ and $y = 3x + 2$
   b. $y = 0.25x + 1$ and $y = 0.75x + 1$
   c. $2y = -4x + 6$ and $4y = -8x + 20$

Assignment

2.1 Imagine that a 50-kg person plans to dance aerobically for 1 hr.
   a. Use the information in Tables 3-3 and 3-5 to design a breakfast that will provide enough energy for this activity.
   b. Write an equation in slope-intercept form that describes the number of kilocalories remaining from the meal in Part **a** at the end of each minute of dancing.
   c. Write an equation in slope-intercept form that describes the number of kilocalories remaining from a 600-kcal meal at the end of each minute of dancing.
   d. Sketch the graphs of both equations on a single set of axes.

2.2 a. Identify the y-intercept of a nonvertical line that passes through the origin (0,0).
   b. Write an equation for the line with a slope of 3 and a y-intercept of 4.
   c. Write an equation for the line that crosses the y-axis at (0,5) and has a slope of −2.
   d. Write an equation for the line with a slope of 7/3 and a y-intercept of 2/5.
   e. Write an equation for the line that crosses the y-axis at (0,−3) and has a slope of 2/5.

2.3 The following two equations were rewritten in slope-intercept form by solving for y in terms of x.

$$\begin{aligned} y - 5 &= 7x \\ y - 5 + 5 &= 7x + 5 \\ y &= 7x + 5 \end{aligned} \qquad \begin{aligned} y + 5x &= 7x \\ y + 5x + (-5x) &= 7x + (-5x) \\ y &= 7x + (-5x) \\ y &= 2x \end{aligned}$$

Use similar methods to solve each of the following equations for y.
   a. $y + 3 = 2x$
   b. $y - 5 = 3x + 2$
   c. $y + 6x = 2x - 7$
   d. $3x + 4y = 7$
   e. $2x - 3y = 6$

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2.4 The following two equations were rewritten in slope-intercept form by multiplying both sides of each equation by the same quantity.

$$\frac{y}{3} = 4x \qquad 4y = 16x$$

$$3 \cdot \frac{y}{3} = 3 \cdot 4x \qquad \frac{1}{4} \cdot 4y = \frac{1}{4} \cdot 16x$$

$$y = 12x \qquad y = 4x$$

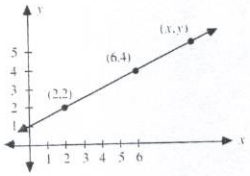
Use a similar method to solve each of the following equations for y.

a. $\frac{y}{-5} = 2x$

b. $7y = 4x$

c. $-2y = 8x$

2.5 As shown in the graph below, the points with coordinates (2,2), (6,4), and (x,y) are on the same line. In this case, the coordinates (x,y) represent any point on the line.



a. Calculate the slope of the line using the points (2,2) and (6,4).

b. Calculate the slope of the line using the points (2,2) and (x,y).

c. Write a mathematical equation that describes the relationship between the two slopes calculated in Parts a and b.

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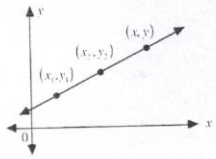
2.6 As shown in the following graph, the points with coordinates (x,y), (x_1,y_1), and (x_2,y_2) are on the same line. In this case, the coordinates (x,y) represent any point on the line.



a. Write a representation for the slope of the line using the points (x_1,y_1), and (x_2,y_2).

b. Write a representation for the slope of the line using the points (x,y) and (x_1,y_1).

c. What relationship exists between the two representations you wrote in Parts a and b?

d. Write a mathematical equation that describes this relationship.

mathematics note

The equation of a line that passes through the point (x_1,y_1) and has a slope of m can be written in the form: $y - y_1 = m(x - x_1)$. This is the **point-slope form** of the equation of a line.

For example, the point-slope equation of a line that passes through the point (2,–4) and has slope of –7 is $y - (-4) = -7(x - 2)$.

2.7 a. Write an equation in point-slope form for each of the following lines.

1. The line that passes through (5,8) and has a slope of –3.
2. The line that passes through (2,10) and (12,5).
3. The line that passes through the origin and has a slope of 2/3.
4. The line that passes through (–6,4) and (–2,5).

b. Rewrite each equation from Part a in slope-intercept form.

* * * * *

2.2 **a.** 0

b. $y = 3x + 4$

c. $y = -2x + 5$

d. $y = \frac{7}{3}x + \frac{2}{5}$

e. $y = \frac{2}{5}x - 3$

2.3 **a.** $y = 2x - 3$

b. $y = 3x + 7$

c. $y = -4x - 7$

d. $y = -\frac{3}{4}x + \frac{7}{4}$

e. $y = \frac{2}{3}x - 2$

2.4 **a.** $y = -10x$

b. $y = \frac{4}{7}x$

c. $y = -4x$

2.5 **a.** $\frac{4-2}{6-2} = \frac{1}{2}$

b. $\frac{y-2}{x-2}$

c. $\frac{y-2}{x-2} = \frac{1}{2}$ or $y - 2 = \frac{1}{2}(x - 2)$

2.6 **a.** $\frac{y_2 - y_1}{x_2 - x_1}$

b. $\frac{y - y_1}{x - x_1}$

c. Sample response: These two expressions are equal because they both represent the slope of the same line.

d. $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$ or $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

2.7 **a.** **1.** $y - 8 = -3(x - 5)$

2. $y - 10 = -\frac{1}{2}(x - 2)$ or $y - 5 = -\frac{1}{2}(x - 12)$

3. $y - 0 = \frac{2}{3}(x - 0)$ or $y = \frac{2}{3}x$

4. $y - 4 = \frac{1}{4}(x - (-6))$ or $y - 5 = \frac{1}{4}(x - (-2))$

b. **1.** $y = -3x + 23$

2. $y = -\frac{1}{2}x + 11$

3. $y = \frac{2}{3}x$

4. $y = \frac{1}{4}x + \frac{11}{2}$

* * * * *