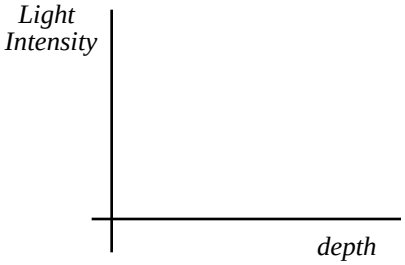
Have you ever noticed how the amount of light differs the further you are under water? Consider the environment of the dolphins pictured below and how the light intensity changes from near the surface to the bottom of the ocean.

*Two Friendly, But Slightly Shy, Dolphins*
*Pose for a Underwater Snapshot*

1. Based on the picture above, answer the following questions.




(a) How does the light change as the depth increases? Sketch a possible graph of the (*depth, light intensity*) relationship that you described.

(b) What accounts for the change in the light intensity?

(c) Suppose you are 10 feet under water, what environmental factors determine the light intensity at 10 feet? If you descend to 15 feet, what determines the light intensity at 15 feet? At 20 feet? Which of these factors remained constant and which changed?

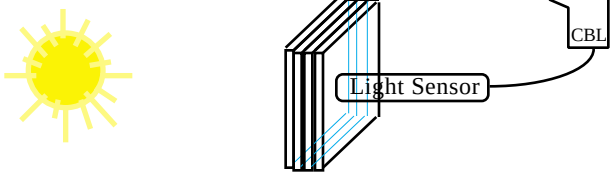
(d) If $I(d)$ is the light intensity at a depth $d$, what does the quantity $I(11) - I(10)$ represent?

(e) How would the quantity $I(12) - I(11)$ compare to $I(11) - I(10)$? What accounts for the difference?

## Experiment 1

The goal of this experiment is to investigate how the intensity of light changes with depth. Have a steady light source such as the light from a window or from a flashlight. Connect a light sensor to a CBL. Do not connect the CBL to a graphing calculator. To take a reading press the mode button on the CBL. You should see the word "Sampling" flash on and off. When you are not taking a reading, press mode or ON to save power.

To model incremental depths, layers of tinted Plexiglas will be used as layers of water. Take a reading with no light on the sensor (cover the sensor with your hand). Take a reading directly from the light source. Add a layer of Plexiglas between the light source and the sensor. Record the new depth of 1 and the new reading. Repeat this process increasing the depth by 1 each time for as many layers as you can.




2. Record your data into a table with three columns like the one below.

| depth ($d$) | Light Intensity $I(d)$ | $I(d+1)–I(d)$ |
|---|---|---|
| | | |

(a) Graph the light intensity as a function of depth. Do all the data points seem reasonable and follow a common curve? If not, are there data points you think should be removed? Explain how the graph compares to your conjecture in question 1 (a).

(b) Using $I(d)$ to represent the light intensity at the current layer of Plexiglas and $I(d+1)$ to represent the light intensity at the next layer, calculate the difference in light intensity between consecutive layers of Plexiglas $I(d+1)–I(d)$. Record the difference in the third column of your table.

(c) Plot $I(d+1)–I(d)$ against the depth $d$. Also plot $I(d+1)–I(d)$ against the light intensity $I(d)$. Explain why an equation of $I(d+1)–I(d)$ vs. $I(d)$ may be easier to find than the other two.

(d) Find an equation for $I(d+1)–I(d)$ vs. $I(d)$. Solve your expression for $I(d+1)$. The resulting equation for $I(d+1)$ is a recurrence relation. Make sense of your equation by explaining

what each part of the equation represents and why the numbers are the size they are and why they have the sign they have.

(e) Plot this recurrence relation on the same axes as the data. What initial value should be used for the recurrence relation? How does the model fit the data? What explanation can you give for any deviation?

(f) Find a general expression for $I(d)$ in terms of initial value $I(0)$. Begin by using your recurrence relation to write $I(1)$ in terms of the $I(0)$. Express $I(2)$ using $I(1)$. Substitute your first expression for $I(1)$ to express $I(2)$ in terms of $I(0)$. Express $I(3)$ in terms of $I(2)$ and then in terms of $I(0)$.

(g) Write a short summary explaining your answers to the following questions:

(i) How does the light intensity relate to the number of layers of Plexiglas (depth)?

(ii) Given a set of data, in what ways could you use the data to test if a function similar to the light intensity function would model the data?

## Experiment 2

The goal of this experiment is to investigate how the intensity of light changes with depth in water. At the bottom of a tube, place a light sensor connected to the CBL. Do not connect the CBL to a graphing calculator. Before recording any light intensity readings, choose a method you will use to record the readings given by the CBL. You will need to decide what reading to record. Some possibilities are to note the maximum, minimum, average or most frequent reading. Place a light source at the top of the tube. Take a reading with no water in the tube and no light in the tube (cover the top). Take a reading with no water but with your light source. Add a fixed amount of water of your choosing. Repeat to gather readings until the top of the tube is reached. You will want to gather about 10 readings. Your light source may initially overwhelm the sensor. The first couple readings may be questionable.

3. Record and analyze your data to find a mathematical model for the light intensity.

| depth ($d$) | Light Intensity $I(d)$ | $I(d+1)–I(d)$ |
|---|---|---|
| | | |

(a) Graph the light intensity as a function of depth. Do all the data points seem reasonable and follow a common curve? If not, are there data points you think should be removed? Explain how the graph compares to your conjecture in question 1 (a).

(b) Using $I(d)$ to represent the light intensity at the current depth of water and $I(d+1)$ to represent the light intensity at the next layer, calculate the difference in light intensity between consecutive layers of water $I(d+1)–I(d)$. Record the difference in the third column of your table.

(c) Plot $I(d+1)–I(d)$ against the depth $d$. Also plot $I(d+1)–I(d)$ against the light intensity $I(d)$. Explain why an equation of $I(d+1)–I(d)$ vs. $I(d)$ may be easier to find than the other two.

(d) Find an equation for $I(d+1)–I(d)$ vs. $I(d)$. Solve your expression for $I(d+1)$. The resulting equation for $I(d+1)$ is a recurrence relation. Make sense of your equation by explaining what each part of the equation represents and why the numbers are the size they are and why they have the sign they have.

(e) Plot this recurrence relation on the same axes as the data. What initial value should be used for the recurrence relation? How does the model fit the data? What explanation can you give for any deviation?

(f) Find a general expression for $I(\mathrm{d})$ in terms of initial value $I(0)$. Use your recurrence relation to write $I(1)$ in terms of the $I(0)$. Express $I(2)$ using $I(1)$. Substitute your first expression for $I(1)$ to express $I(2)$ in terms of $I(0)$. Express $I(3)$ in terms of $I(2)$ and then in terms of $I(0)$. Use these results to determine a formula for $I(\mathrm{d})$.

(g) Write a short summary explaining your answers to the following questions.

(i) How does the light intensity relate to the number of layers of water (depth)?

(ii) Given a set of data, in what ways could you use the data to test if a function similar to the light intensity function would model the data?

(iii) How are the methods and results of Experiment 1 and Experiment 2 similar, and how they are different?