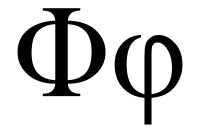
**History of the Golden Ratio**

[](http://en.wikipedia.org/wiki/File:Phi_uc_lc.svg)

[http://bits.wikimedia.org/skins-1.5/common/images/magnify-clip.png](http://en.wikipedia.org/wiki/File:Phi_uc_lc.svg)

Mathematician [Mark Barr](http://en.wikipedia.org/wiki/Mark_Barr) proposed using the first letter in the name of Greek sculptor [Phidias](http://en.wikipedia.org/wiki/Phidias), *phi*, to symbolize the golden ratio. Usually, the lowercase form (φ) is used. Sometimes, the uppercase form (Φ) is used for the [reciprocal](http://en.wikipedia.org/wiki/Reciprocal_(mathematics)) of the golden ratio, 1/φ.[[10]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-MathWorld_GR_Conjugate-9)

[](http://en.wikipedia.org/wiki/File:Michael_Maestlin.jpg)

[http://bits.wikimedia.org/skins-1.5/common/images/magnify-clip.png](http://en.wikipedia.org/wiki/File:Michael_Maestlin.jpg)

[Michael Maestlin](http://en.wikipedia.org/wiki/Michael_Maestlin), first to publish a decimal approximation of the golden ratio, in 1597.

The golden ratio has fascinated Western intellectuals of diverse interests for at least 2,400 years:

Some of the greatest mathematical minds of all ages, from [Pythagoras](http://en.wikipedia.org/wiki/Pythagoras) and [Euclid](http://en.wikipedia.org/wiki/Euclid) in [ancient Greece](http://en.wikipedia.org/wiki/Ancient_Greece), through the medieval Italian mathematician [Leonardo of Pisa](http://en.wikipedia.org/wiki/Fibonacci) and the Renaissance astronomer [Johannes Kepler](http://en.wikipedia.org/wiki/Johannes_Kepler), to present-day scientific figures such as Oxford physicist [Roger Penrose](http://en.wikipedia.org/wiki/Roger_Penrose), have spent endless hours over this simple ratio and its properties. But the fascination with the Golden Ratio is not confined just to mathematicians. Biologists, artists, musicians, historians, architects, psychologists, and even mystics have pondered and debated the basis of its ubiquity and appeal. In fact, it is probably fair to say that the Golden Ratio has inspired thinkers of all disciplines like no other number in the history of mathematics.

—[Mario Livio](http://en.wikipedia.org/wiki/Mario_Livio), *The Golden Ratio: The Story of Phi, The World's Most Astonishing Number*[*[page needed](http://en.wikipedia.org/wiki/Wikipedia:Citing_sources" \o "Wikipedia:Citing sources)*]

[Ancient Greek](http://en.wikipedia.org/wiki/Ancient_Greece) mathematicians first studied what we now call the golden ratio because of its frequent appearance in [geometry](http://en.wikipedia.org/wiki/Geometry). The division of a line into "extreme and mean ratio" (the golden section) is important in the geometry of regular [pentagrams](http://en.wikipedia.org/wiki/Pentagram#Geometry) and [pentagons](http://en.wikipedia.org/wiki/Pentagon). The Greeks usually attributed discovery of this concept to [Pythagoras](http://en.wikipedia.org/wiki/Pythagoras) or his [followers](http://en.wikipedia.org/wiki/Pythagoreanism). The regular pentagram, which has a regular pentagon inscribed within it, was the Pythagoreans' symbol.

[Euclid](http://en.wikipedia.org/wiki/Euclid)'s [*Elements*](http://en.wikipedia.org/wiki/Euclid%27s_Elements) ([Greek](http://en.wikipedia.org/wiki/Greek_language): Στοιχεῖα) provides the first known written definition of what is now called the golden ratio: "A straight line is said to have been *cut in extreme and mean ratio* when, as the whole line is to the greater segment, so is the greater to the less."[[5]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-Elements_6.3-4) Euclid explains a construction for cutting (sectioning) a line "in extreme and mean ratio", i.e. the golden ratio.[[11]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-10) Throughout the *Elements*, several propositions ([theorems](http://en.wikipedia.org/wiki/Theorem) in modern terminology) and their proofs employ the golden ratio.[[12]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-11) Some of these propositions show that the golden ratio is an [irrational number](http://en.wikipedia.org/wiki/Irrational_number).

The name "extreme and mean ratio" was the principal term used from the 3rd century BC[[5]](http://en.wikipedia.org/wiki/Golden_ratio" \l "cite_note-Elements_6.3-4) until about the 18th century.

The modern history of the golden ratio starts with [Luca Pacioli](http://en.wikipedia.org/wiki/Luca_Pacioli)'s [*De divina proportione*](http://en.wikipedia.org/wiki/De_divina_proportione) of 1509, which captured the imagination of artists, architects, scientists, and mystics with the properties, mathematical and otherwise, of the golden ratio.

The first known approximation of the (inverse) golden ratio by a [decimal fraction](http://en.wikipedia.org/wiki/Decimal_fraction), stated as "about 0.6180340," was written in 1597 by Prof. [Michael Maestlin](http://en.wikipedia.org/wiki/Michael_Maestlin) of the [University of Tübingen](http://en.wikipedia.org/wiki/University_of_T%C3%BCbingen) in a letter to his former student [Johannes Kepler](http://en.wikipedia.org/wiki/Johannes_Kepler).[[13]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-12)

Since the twentieth century, the golden ratio has been represented by the [Greek letter](http://en.wikipedia.org/wiki/Greek_alphabet) ***Φ*** or ***φ*** ([phi](http://en.wikipedia.org/wiki/Phi), after [Phidias](http://en.wikipedia.org/wiki/Phidias), a sculptor who is said to have employed it) or less commonly by ***τ*** ([tau](http://en.wikipedia.org/wiki/Tau), the first letter of the [ancient Greek](http://en.wikipedia.org/wiki/Ancient_Greek) root τομή—meaning *cut*).[[14]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-13)[[2]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-livio-1)

**[**[**edit**](http://en.wikipedia.org/w/index.php?title=Golden_ratio&action=edit&section=3)**] Timeline**

Timeline according to Priya Hemenway[[15]](http://en.wikipedia.org/wiki/Golden_ratio" \l "cite_note-Hemenway.2CP-14).

* [Phidias](http://en.wikipedia.org/wiki/Phidias) (490–430 BC) made the [Parthenon](http://en.wikipedia.org/wiki/Parthenon) statues that seem to embody the golden ratio.
* [Plato](http://en.wikipedia.org/wiki/Plato) (427–347 BC), in his [*Timaeus*](http://en.wikipedia.org/wiki/Timaeus_(dialogue)), describes five possible regular solids (the [Platonic solids](http://en.wikipedia.org/wiki/Platonic_solids): the [tetrahedron](http://en.wikipedia.org/wiki/Tetrahedron), [cube](http://en.wikipedia.org/wiki/Cube), [octahedron](http://en.wikipedia.org/wiki/Octahedron), [dodecahedron](http://en.wikipedia.org/wiki/Dodecahedron) and [icosahedron](http://en.wikipedia.org/wiki/Icosahedron)), some of which are related to the golden ratio.[[16]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-15)
* [Euclid](http://en.wikipedia.org/wiki/Euclid) (c. 325–c. 265 BC), in his [*Elements*](http://en.wikipedia.org/wiki/Euclid%27s_Elements), gave the first recorded definition of the golden ratio, which he called, as translated into English, "extreme and mean ratio" (Greek: ἄκρος καὶ μέσος λόγος).[[5]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-Elements_6.3-4)
* [Fibonacci](http://en.wikipedia.org/wiki/Fibonacci) (1170–1250) mentioned the [numerical series](http://en.wikipedia.org/wiki/Sequence) now named after him in his [*Liber Abaci*](http://en.wikipedia.org/wiki/Liber_Abaci); the ratio of sequential elements of the [Fibonacci sequence](http://en.wikipedia.org/wiki/Fibonacci_number) approaches the golden ratio asymptotically.
* [Luca Pacioli](http://en.wikipedia.org/wiki/Luca_Pacioli) (1445–1517) defines the golden ratio as the "divine proportion" in his *Divina Proportione*.
* [Johannes Kepler](http://en.wikipedia.org/wiki/Johannes_Kepler) (1571–1630) proves that the golden ratio is the limit of the ratio of consecutive Fibonacci numbers,[[17]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-tatt-16) and describes the golden ratio as a "precious jewel": "Geometry has two great treasures: one is the [Theorem of Pythagoras](http://en.wikipedia.org/wiki/Pythagorean_theorem), and the other the division of a line into extreme and mean ratio; the first we may compare to a measure of gold, the second we may name a precious jewel." These two treasures are combined in the [Kepler triangle](http://en.wikipedia.org/wiki/Kepler_triangle).
* [Charles Bonnet](http://en.wikipedia.org/wiki/Charles_Bonnet) (1720–1793) points out that in the spiral [phyllotaxis](http://en.wikipedia.org/wiki/Phyllotaxis) of plants going [clockwise](http://en.wikipedia.org/wiki/Clockwise) and counter-clockwise were frequently two successive Fibonacci series.
* [Martin Ohm](http://en.wikipedia.org/wiki/Martin_Ohm) (1792–1872) is believed to be the first to use the term *goldener Schnitt* (golden section) to describe this ratio, in 1835.[[18]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-17)
* [Edouard Lucas](http://en.wikipedia.org/wiki/Edouard_Lucas) (1842–1891) gives the numerical sequence now known as the Fibonacci sequence its present name.
* Mark Barr (20th century) suggests the Greek letter phi (**φ**), the initial letter of Greek sculptor Phidias's name, as a [symbol](http://en.wikipedia.org/wiki/Symbol) for the golden ratio.[[19]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-18)
* [Roger Penrose](http://en.wikipedia.org/wiki/Roger_Penrose) (b.1931) discovered a symmetrical pattern that uses the golden ratio in the field of [aperiodic tilings](http://en.wikipedia.org/wiki/Aperiodic_tiling), which led to new discoveries about [quasicrystals](http://en.wikipedia.org/wiki/Quasicrystals).

**[**[**edit**](http://en.wikipedia.org/w/index.php?title=Golden_ratio&action=edit&section=4)**] Aesthetics**

*Further information:* [*List of works designed with the golden ratio*](http://en.wikipedia.org/wiki/List_of_works_designed_with_the_golden_ratio)

Beginning in the [Renaissance](http://en.wikipedia.org/wiki/Renaissance), a body of literature on the [aesthetics](http://en.wikipedia.org/wiki/Aesthetics) of the golden ratio was developed. As a result, architects, artists, book designers, and others have been encouraged to use the golden ratio in the dimensional relationships of their works.

The first and most influential of these was *De Divina Proportione* by [Luca Pacioli](http://en.wikipedia.org/wiki/Luca_Pacioli), a three-volume work published in 1509. Pacioli, a [Franciscan](http://en.wikipedia.org/wiki/Franciscan) [friar](http://en.wikipedia.org/wiki/Friar), was known mostly as a mathematician, but he was also trained and keenly interested in art. *De Divina Proportione* explored the mathematics of the golden ratio. Though it is often said that Pacioli advocated the golden ratio's application to yield pleasing, harmonious proportions, Livio points out that that interpretation has been traced to an error in 1799, and that Pacioli actually advocated the [Vitruvian](http://en.wikipedia.org/wiki/Vitruvius) system of rational proportions.[[2]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-livio-1) Pacioli also saw Catholic religious significance in the ratio, which led to his work's title. Containing illustrations of regular solids by [Leonardo Da Vinci](http://en.wikipedia.org/wiki/Leonardo_Da_Vinci), Pacioli's longtime friend and collaborator, *De Divina Proportione* was a major influence on generations of artists and architects alike.

**[**[**edit**](http://en.wikipedia.org/w/index.php?title=Golden_ratio&action=edit&section=5)**] Architecture**

[](http://en.wikipedia.org/wiki/File:Acropolis_of_Athens_01361.JPG)

[http://bits.wikimedia.org/skins-1.5/common/images/magnify-clip.png](http://en.wikipedia.org/wiki/File:Acropolis_of_Athens_01361.JPG)

Many of the proportions of [The Parthenon](http://en.wikipedia.org/wiki/The_Parthenon) are golden ratios.

Some studies of the [Acropolis](http://en.wikipedia.org/wiki/Acropolis), including the [Parthenon](http://en.wikipedia.org/wiki/Parthenon), conclude that many of its proportions approximate the golden ratio. The Parthenon's facade as well as elements of its facade and elsewhere are said to be circumscribed by golden rectangles.[[20]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-19) To the extent that classical buildings or their elements are proportioned according to the golden ratio, this might indicate that their architects were aware of the golden ratio and consciously employed it in their designs. Alternatively, it is possible that the architects used their own sense of good proportion, and that this led to some proportions that closely approximate the golden ratio. On the other hand, such retrospective analyses can always be questioned on the ground that the investigator chooses the points from which measurements are made or where to superimpose golden rectangles, and that these choices affect the proportions observed.

Some scholars deny that the Greeks had any aesthetic association with golden ratio. For example, Midhat J. Gazalé says, "It was not until Euclid, however, that the golden ratio's mathematical properties were studied. In the *Elements* (308 BC) the Greek mathematician merely regarded that number as an interesting irrational number, in connection with the middle and extreme ratios. Its occurrence in regular pentagons and decagons was duly observed, as well as in the dodecahedron (a [regular polyhedron](http://en.wikipedia.org/wiki/Regular_polyhedron) whose twelve faces are regular pentagons). It is indeed exemplary that the great Euclid, contrary to generations of mystics who followed, would soberly treat that number for what it is, without attaching to it other than its factual properties."[[21]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-20) And [Keith Devlin](http://en.wikipedia.org/wiki/Keith_Devlin) says, "Certainly, the oft repeated assertion that the Parthenon in Athens is based on the golden ratio is not supported by actual measurements. In fact, the entire story about the Greeks and golden ratio seems to be without foundation. The one thing we know for sure is that Euclid, in his famous textbook *Elements*, written around 300 BC, showed how to calculate its value."[[22]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-21) Near-contemporary sources like [Vitruvius](http://en.wikipedia.org/wiki/Vitruvius) exclusively discuss proportions that can be expressed in whole numbers, i.e. commensurate as opposed to irrational proportions.

A geometrical analysis of the [Great Mosque of Kairouan](http://en.wikipedia.org/wiki/Mosque_of_Oqba) reveals a consistent application of the golden ratio throughout the design, according to Boussora and Mazouz.[[23]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-22) It is found in the overall proportion of the plan and in the dimensioning of the prayer space, the court, and the [minaret](http://en.wikipedia.org/wiki/Minaret). Boussora and Mazouz also examined earlier archaeological theories about the mosque, and demonstrate the geometric constructions based on the golden ratio by applying these constructions to the plan of the mosque to test their hypothesis.

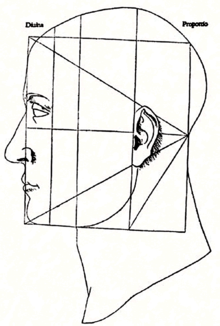
The Swiss [architect](http://en.wikipedia.org/wiki/Architect) [Le Corbusier](http://en.wikipedia.org/wiki/Le_Corbusier), famous for his contributions to the [modern](http://en.wikipedia.org/wiki/Modernism) [international style](http://en.wikipedia.org/wiki/International_style_(architecture)), centered his design philosophy on systems of harmony and proportion. Le Corbusier's faith in the mathematical order of the universe was closely bound to the golden ratio and the Fibonacci series, which he described as "rhythms apparent to the eye and clear in their relations with one another. And these rhythms are at the very root of human activities. They resound in man by an organic inevitability, the same fine inevitability which causes the tracing out of the Golden Section by children, old men, savages and the learned."[[24]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-23)

Le Corbusier explicitly used the golden ratio in his [Modulor](http://en.wikipedia.org/wiki/Modulor) system for the [scale](http://en.wikipedia.org/wiki/Scale_(ratio)) of [architectural proportion](http://en.wikipedia.org/wiki/Proportion_(architecture)). He saw this system as a continuation of the long tradition of [Vitruvius](http://en.wikipedia.org/wiki/Vitruvius), Leonardo da Vinci's "[Vitruvian Man](http://en.wikipedia.org/wiki/Vitruvian_Man" \o "Vitruvian Man)", the work of [Leon Battista Alberti](http://en.wikipedia.org/wiki/Leon_Battista_Alberti), and others who used the proportions of the human body to improve the appearance and function of [architecture](http://en.wikipedia.org/wiki/Architecture). In addition to the golden ratio, Le Corbusier based the system on [human measurements](http://en.wikipedia.org/wiki/Anthropometry), [Fibonacci numbers](http://en.wikipedia.org/wiki/Fibonacci_numbers), and the double unit. He took Leonardo's suggestion of the golden ratio in human proportions to an extreme: he sectioned his model human body's height at the navel with the two sections in golden ratio, then subdivided those sections in golden ratio at the knees and throat; he used these golden ratio proportions in the [Modulor](http://en.wikipedia.org/wiki/Modulor) system. Le Corbusier's 1927 Villa Stein in [Garches](http://en.wikipedia.org/wiki/Garches) exemplified the Modulor system's application. The villa's rectangular ground plan, elevation, and inner structure closely approximate golden rectangles.[[25]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-24)

Another Swiss architect, [Mario Botta](http://en.wikipedia.org/wiki/Mario_Botta), bases many of his designs on geometric figures. Several private houses he designed in Switzerland are composed of squares and circles, cubes and cylinders. In a house he designed in [Origlio](http://en.wikipedia.org/wiki/Origlio), the golden ratio is the proportion between the central section and the side sections of the house.[[26]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-25)

In a recent book, author Jason Elliot speculated that the golden ratio was used by the designers of the [Naqsh-e Jahan Square](http://en.wikipedia.org/wiki/Naqsh-e_Jahan_Square) and the adjacent Lotfollah mosque.[[27]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-26)

**[**[**edit**](http://en.wikipedia.org/w/index.php?title=Golden_ratio&action=edit&section=6)**] Painting**

[](http://en.wikipedia.org/wiki/File:Divina_proportione.png)

[http://bits.wikimedia.org/skins-1.5/common/images/magnify-clip.png](http://en.wikipedia.org/wiki/File:Divina_proportione.png)

Illustration from Luca Pacioli's *De Divina Proportione* applies geometric proportions to the human face.

[Leonardo da Vinci](http://en.wikipedia.org/wiki/Leonardo_da_Vinci)'s illustrations of [polyhedra](http://en.wikipedia.org/wiki/Polyhedra) in *De Divina Proportione* (*On the Divine Proportion*) and his views that some bodily proportions exhibit the golden ratio have led some scholars to speculate that he incorporated the golden ratio in his paintings.[[28]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-27) But the suggestion that his [*Mona Lisa*](http://en.wikipedia.org/wiki/Mona_Lisa), for example, employs golden ratio proportions, is not supported by anything in Leonardo's own writings.[[29]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-28)

[Salvador Dalí](http://en.wikipedia.org/wiki/Salvador_Dal%C3%AD) explicitly used the golden ratio in his masterpiece, [*The Sacrament of the Last Supper*](http://en.wikipedia.org/wiki/The_Sacrament_of_the_Last_Supper). The dimensions of the canvas are a golden rectangle. A huge dodecahedron, with edges in golden ratio to one another, is suspended above and behind Jesus and dominates the composition.[[2]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-livio-1)[[30]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-29)

[Mondrian](http://en.wikipedia.org/wiki/Piet_Mondrian) has been said to have used the golden section extensively in his geometrical paintings,[[31]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-30) though other experts (including critic [Yve-Alain Bois](http://en.wikipedia.org/wiki/Yve-Alain_Bois)) have denied this fact.[[2]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-livio-1)

A statistical study on 565 works of art of different great painters, performed in 1999, found that these artists had not used the golden ratio in the size of their canvases. The study concluded that the average ratio of the two sides of the paintings studied is 1.34, with averages for individual artists ranging from 1.04 (Goya) to 1.46 (Bellini).[[32]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-31) On the other hand, Pablo Tosto listed over 350 works by well-known artists, including more than 100 which have canvasses with golden rectangle and root-5 proportions, and others with proportions like root-2, 3, 4, and 6.[[33]](http://en.wikipedia.org/wiki/Golden_ratio#cite_note-32)