TOWER OF Hanoi—Exploring y= 2n -1

Web link for virtual game.

<http://www.mazeworks.com/hanoi/>

See second page (Student worksheet) for background information and the related table.

SOLUTION…

If the legend were true, and if the priests were able to move disks at a rate of one per second, using the smallest number of moves, it would take them 264−1 seconds or roughly 585 [billion](http://en.wikipedia.org/wiki/1,000,000,000_(number)) years;[[1]](http://en.wikipedia.org/wiki/Tower_of_Hanoi" \l "cite_note-0) it would take 18,446,744,073,709,551,615 turns to finish.

The **Tower of Hanoi** or **Towers of** [**Hanoi**](http://en.wikipedia.org/wiki/Hanoi) is a [mathematical game](http://en.wikipedia.org/wiki/Mathematical_game) or [puzzle](http://en.wikipedia.org/wiki/Puzzle). It consists of three rods, and a number of disks of different sizes which can slide onto any rod. The puzzle starts with the disks in a neat stack in ascending order of size on one rod, the smallest at the top, thus making a conical shape.

|  |
| --- |
| RULES of THE TOWER: The objective of the puzzle is to move the entire stack to another rod, obeying the following rules:   * Only one disk may be moved at a time. * Each move consists of taking the upper disk from one of the rods and sliding it onto another rod, on top of the other disks that may already be present on that rod. * No disk may be placed on top of a smaller disk. |

## Origins

The puzzle was invented by the [French](http://en.wikipedia.org/wiki/French_people) [mathematician](http://en.wikipedia.org/wiki/Mathematician) [Édouard Lucas](http://en.wikipedia.org/wiki/%C3%89douard_Lucas) in 1883. There is a legend about a [Vietnamese](http://en.wikipedia.org/wiki/Vietnam) temple which contains a large room with three time-worn posts in it surrounded by 64 golden disks. The priests of [Hanoi](http://en.wikipedia.org/wiki/Hanoi), acting out the command of an ancient prophecy, have been moving these disks, in accordance with the rules of the puzzle, since that time. The puzzle is therefore also known as the Tower of [Brahma](http://en.wikipedia.org/wiki/Brahma) puzzle. According to the legend, when the last move of the puzzle is completed, the world will end. It is not clear whether Lucas invented this legend or was inspired by it.

There are many variations on this legend. For instance, in some tellings, the temple is a [monastery](http://en.wikipedia.org/wiki/Monastery) and the priests are [monks](http://en.wikipedia.org/wiki/Monk). The temple or monastery may be said to be in different parts of the world — including [Hanoi](http://en.wikipedia.org/wiki/Hanoi), [Vietnam](http://en.wikipedia.org/wiki/Vietnam), and may be associated with any [religion](http://en.wikipedia.org/wiki/Religion). In some versions, other elements are introduced, such as the fact that the tower was created at the beginning of the world, or that the priests or monks may make only one move per day.

The [Flag Tower of Hanoi](http://en.wikipedia.org/wiki/Flag_Tower_of_Hanoi) may have served as the inspiration for the name.

TOWER OF HANOI Investigation Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Using your model “towers” or the web-based version, completely move all disks from one tower to the new tower. You will completely move each set of disks and record the number of moves in the table below. After completing the table, identify a pattern that may serve as the equation for this puzzle.

|  |  |  |
| --- | --- | --- |
| Number of Disks | Number of Moves to Shift Towers | Pattern |
| 1 | 1 |  |
| 2 | 3 |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| .  .  . |  |  |
| N |  |  |

Solve it…

1. Find the minimum number of moves required to move all 64 disks (from the legend).
2. Suppose it takes one second to move one disk. How long would it take the priests to move all 64 disk (according to the rules of the legend)?