

10.3

30°-60°-90° Triangles

Goal

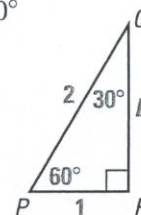
Find the side lengths of 30°-60°-90° triangles.

Key Words

• 30°-60°-90° triangle

A right triangle with angle measures of 30°, 60°, and 90° is called a **30°-60°-90° triangle**.

Activity 10.3 shows that the ratio of the length of the hypotenuse of a 30°-60°-90° triangle to the length of the shorter leg is 2 : 1.



EXAMPLE 1 Find Leg Length

In the diagram above, $\triangle PQR$ is a 30°-60°-90° triangle with $PQ = 2$ and $PR = 1$. Find the value of b .

Solution

You can use the Pythagorean Theorem to find the value of b .

$$(\text{leg})^2 + (\text{leg})^2 = (\text{hypotenuse})^2$$

$$1^2 + b^2 = 2^2$$

$$1 + b^2 = 4$$

$$b^2 = 3$$

$$b = \sqrt{3}$$

Write the Pythagorean Theorem.

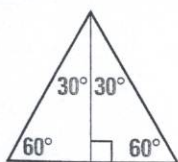
Substitute.

Simplify.

Subtract 1 from each side.

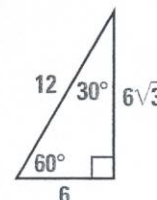
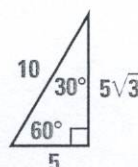
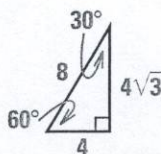
Take the square root of each side.

Visualize It!



An equilateral triangle can be divided into two 30°-60°-90° triangles.

Because all 30°-60°-90° triangles are similar, the ratio of the length of the longer leg to the length of the shorter leg is always $\sqrt{3} : 1$. This result is summarized in the theorem below.

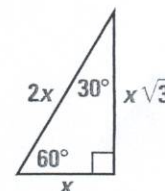


THEOREM 10.2

30°-60°-90° Triangle Theorem

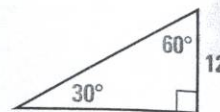
Words In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is the length of the shorter leg times $\sqrt{3}$.

Symbols Hypotenuse = 2 • shorter leg
Longer leg = shorter leg • $\sqrt{3}$



EXAMPLE 2 Find Hypotenuse Length

In the 30° - 60° - 90° triangle at the right, the length of the shorter leg is given. Find the length of the hypotenuse.

**Solution**

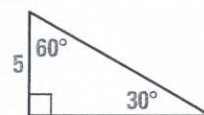
The hypotenuse of a 30° - 60° - 90° triangle is twice as long as the shorter leg.

$$\begin{aligned} \text{hypotenuse} &= 2 \cdot \text{shorter leg} && 30^\circ\text{-}60^\circ\text{-}90^\circ \text{ Triangle Theorem} \\ &= 2 \cdot 12 && \text{Substitute.} \\ &= 24 && \text{Simplify.} \end{aligned}$$

ANSWER ▶ The length of the hypotenuse is 24.

EXAMPLE 3 Find Longer Leg Length

In the 30° - 60° - 90° triangle at the right, the length of the shorter leg is given. Find the length of the longer leg.

**Solution**

The length of the longer leg of a 30° - 60° - 90° triangle is the length of the shorter leg times $\sqrt{3}$.

$$\begin{aligned} \text{longer leg} &= \text{shorter leg} \cdot \sqrt{3} && 30^\circ\text{-}60^\circ\text{-}90^\circ \text{ Triangle Theorem} \\ &= 5 \cdot \sqrt{3} && \text{Substitute.} \end{aligned}$$

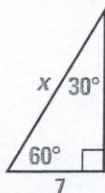
ANSWER ▶ The length of the longer leg is $5\sqrt{3}$.

In a 30° - 60° - 90° triangle, the longer leg is opposite the 60° angle, and the shorter leg is opposite the 30° angle.

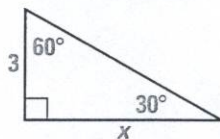
Checkpoint **Find Lengths in a Triangle**

Find the value of x . Write your answer in radical form.

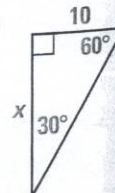
1.



2.

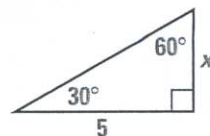


3.



EXAMPLE 4 Find Shorter Leg Length

In the 30° - 60° - 90° triangle at the right, the length of the longer leg is given. Find the length x of the shorter leg. Round your answer to the nearest tenth.

**Solution**

The length of the longer leg of a 30° - 60° - 90° triangle is the length of the shorter leg times $\sqrt{3}$.

$$\text{longer leg} = \text{shorter leg} \cdot \sqrt{3} \quad 30^\circ\text{-}60^\circ\text{-}90^\circ \text{ Triangle Theorem}$$

$$5 = x \cdot \sqrt{3} \quad \text{Substitute.}$$

$$\frac{5}{\sqrt{3}} = x \quad \text{Divide each side by } \sqrt{3}.$$

$$2.9 \approx x \quad \text{Use a calculator.}$$

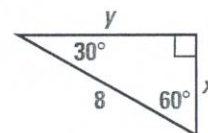
ANSWER ▶ The length of the shorter leg is about 2.9.



MORE EXAMPLES
More examples at
classzone.com

EXAMPLE 5 Find Leg Lengths

In the 30° - 60° - 90° triangle at the right, the length of the hypotenuse is given. Find the length x of the shorter leg and the length y of the longer leg.

**Solution**

Use the 30° - 60° - 90° Triangle Theorem to find the length of the shorter leg. Then use that value to find the length of the longer leg.

Shorter leg

$$\text{hypotenuse} = 2 \cdot \text{shorter leg}$$

$$8 = 2 \cdot x$$

$$4 = x$$

Longer leg

$$\text{longer leg} = \text{shorter leg} \cdot \sqrt{3}$$

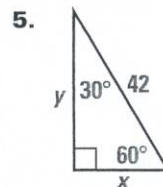
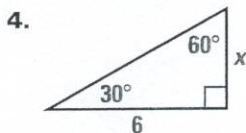
$$y = 4 \cdot \sqrt{3}$$

$$y = 4\sqrt{3}$$

ANSWER ▶ The length of the shorter leg is 4.
The length of the longer leg is $4\sqrt{3}$.

Checkpoint Find Leg Lengths

Find the value of each variable. Round your answer to the nearest tenth.



10.3 Exercises

Guided Practice

Vocabulary Check

1. Name two special right triangles by their angle measures.

Skill Check

Use the diagram to tell whether the equation is *true* or *false*.

2. $t = 7\sqrt{3}$

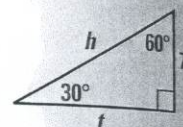
3. $t = \sqrt{3}h$

4. $h = 2t$

5. $h = 14$

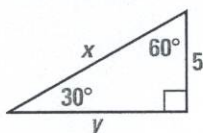
6. $7 = \frac{h}{2}$

7. $7 = \frac{t}{\sqrt{3}}$

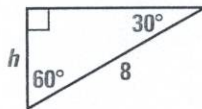


Find the value of each variable. Write your answers in radical form.

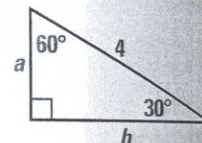
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10.



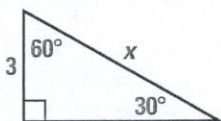
Practice and Applications

Extra Practice

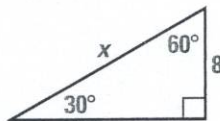
See p. 693.

Finding Hypotenuse Lengths Find the length of the hypotenuse.

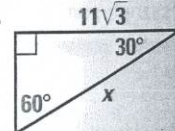
11.



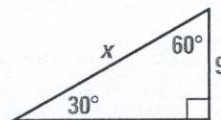
12.



13.



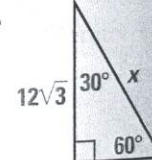
14.



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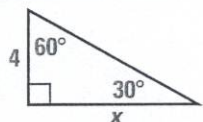


16.

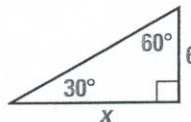


Finding Leg Lengths Find the length of the longer leg of the triangle. Write your answer in radical form.

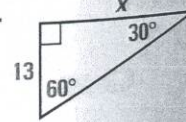
17.



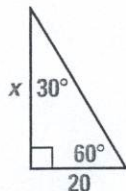
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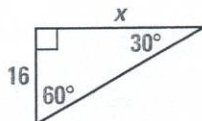
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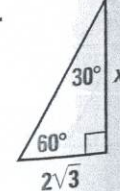
20.



21.



22.



Homework Help

Example 2: Exs. 11–16

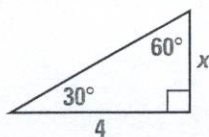
Example 3: Exs. 17–22

Example 4: Exs. 23–28

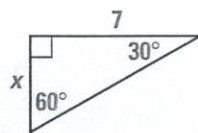
Example 5: Exs. 29–35

Finding Leg Lengths Find the length of the shorter leg of the triangle. Round your answer to the nearest tenth.

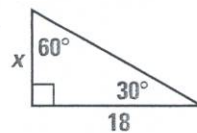
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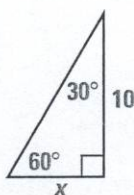
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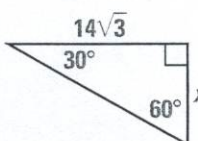
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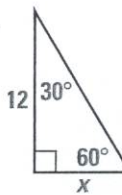
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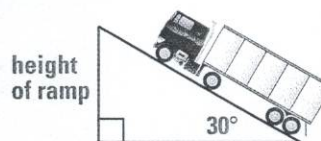
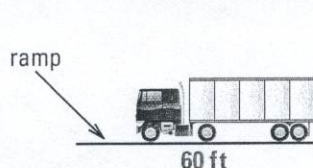
27.



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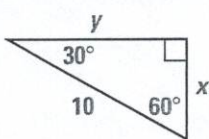


29. Tipping Platform A tipping platform is a ramp used to unload trucks as shown below. What is the height of an 60 foot ramp when it is tipped up to a 30° angle?

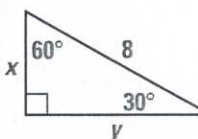


Finding Leg Lengths Find the value of each variable. Write your answers in radical form.

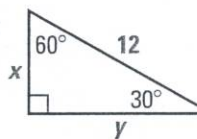
30.



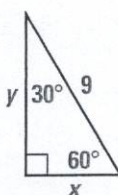
31.



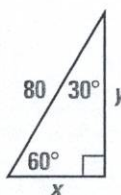
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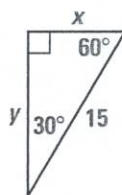
33.



34.



35.



36. Fitness A "crunch" is the type of sit-up shown in the photo at the right. A personal trainer tells you that in doing a crunch, your back and shoulders should be lifted to an angle of about 30° . If your shoulder-to-waist length is 18 inches, how high should your shoulders be lifted?



Link to Careers

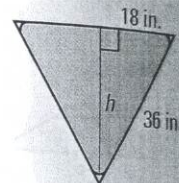


PERSONAL TRAINERS develop fitness programs suited to an individual's abilities and goals. They study anatomy, nutrition, and physiology.

Career Links
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EXAMPLE Find Area Using 30°-60°-90° Triangles

The road sign is shaped like an equilateral triangle with side lengths of 36 inches. Estimate the area of the sign.

**Solution**

Divide the triangle into two 30°-60°-90° triangles.

The length of the shorter leg of each triangle is 18 inches. The length of the longer leg of each triangle is $18\sqrt{3}$ inches, by the 30°-60°-90° Triangle Theorem.

Use the formula for the area of a triangle.

$$\text{Area} = \frac{1}{2}bh = \frac{1}{2} \cdot 36 \cdot 18\sqrt{3} \approx 561.18$$

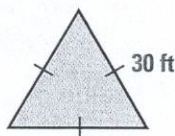
ANSWER ▶ The area of the sign is about 561 square inches.

Student Help**LOOK BACK**

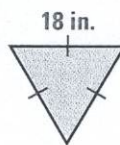
To review the area of a triangle, see p. 431.

Finding Area Find the area of each triangle. Use the example above as a model.

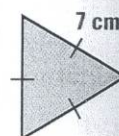
37.



38.



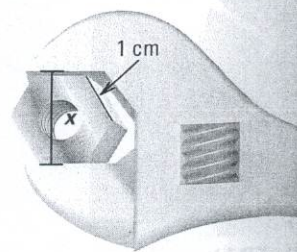
39.



40. **Using Algebra** Use the figure in the example above to explain why the area A of an equilateral triangle with side length s is given by the formula $A = \frac{1}{4} \cdot s^2 \cdot \sqrt{3}$.

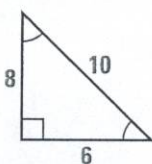
41. **Visualize It!** A 30°-60°-90° triangle has a shorter leg length of 15 centimeters. Sketch the triangle and find the length of the hypotenuse and the length of the longer leg in radical form.

42. **Challenge** The side length of the hexagonal nut shown at the right is 1 centimeter. Find the value of x . (Hint: Use the fact that a regular hexagon can be divided into six congruent equilateral triangles.)

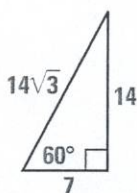
**Standardized Test Practice**

43. **Multiple Choice** Which triangle is labeled correctly?

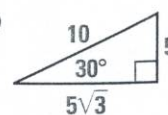
(A)



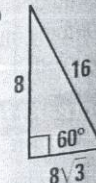
(B)



(C)

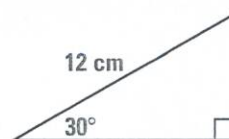


(D)



- 44. Multiple Choice** Find the perimeter of the triangle shown below to the nearest tenth of a centimeter.

- (F) 28.4 cm (G) 30 cm
(H) 31.2 cm (J) 41.6 cm

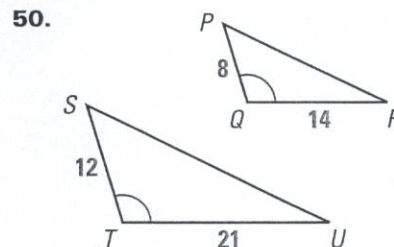
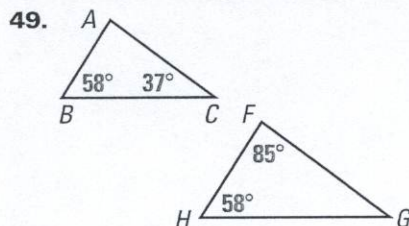


Mixed Review

Writing Ratios A football team won 10 games and lost 6 games. Find the ratio. (Lesson 7.1)

45. wins to losses 46. losses to wins
47. wins to the number of games 48. losses to the number of games

Similar Triangles Determine whether the triangles are similar. If they are similar, write a similarity statement. (Lessons 7.3, 7.4)



Algebra Skills

Evaluating Expressions Evaluate the expression when $x = -4$. (Skills Review, p. 670)

51. $5x + 4$ 52. $10x - 1$ 53. $x^2 - 7$
54. $(x + 3)(x - 3)$ 55. $2x^2 - x + 1$ 56. $5x^2 + 2x - 3$

Quiz 1

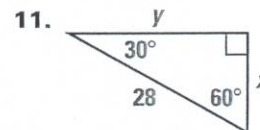
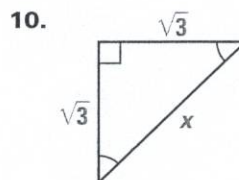
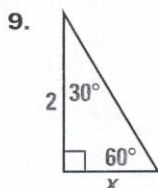
Multiply the radical expression. Then simplify if possible. (Lesson 10.1)

1. $\sqrt{8} \cdot \sqrt{3}$ 2. $\sqrt{2} \cdot \sqrt{15}$ 3. $\sqrt{8} \cdot \sqrt{18}$ 4. $\sqrt{80} \cdot \sqrt{5}$

Simplify the radical expression. (Lesson 10.1)

5. $\sqrt{27}$ 6. $\sqrt{176}$ 7. $\sqrt{52}$ 8. $\sqrt{180}$

Find the value of each variable. Write your answer in radical form. (Lessons 10.2, 10.3)



Materials

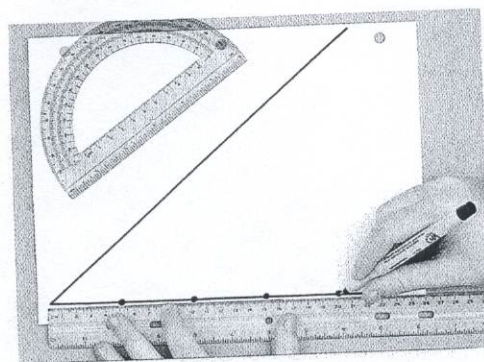
- centimeter ruler
- protractor
- calculator

Question

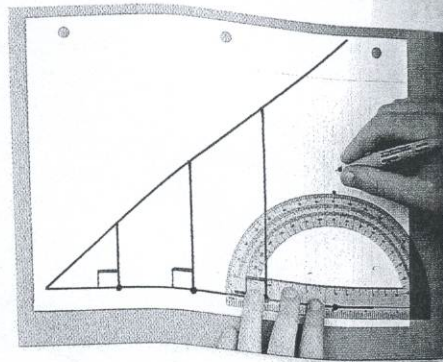
Does the size of similar right triangles affect the ratio of their leg lengths?

Explore

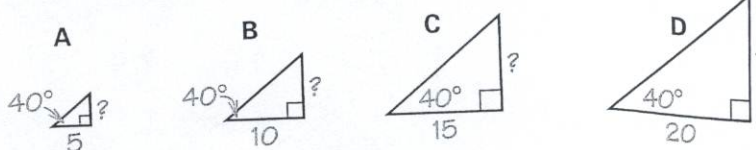
- ① Draw a 40° angle. Mark points every 5 cm along one side.



- ② Draw perpendicular segments through four of the points.

**Think About It**

1. There are four similar triangles in your drawing. Measure the legs and complete a table like the one below.



Triangle	longer leg	shorter leg	$\frac{\text{shorter leg}}{\text{longer leg}}$
A	5 cm	4.2 cm	0.84
B	10 cm	?	?
C	15 cm	?	?
D	20 cm	?	?

2. Compare the ratios of the leg lengths in the last column of your table. What do you notice?
3. Repeat using a different acute angle. Based on your results, does the ratio of leg lengths depend on the size of a right triangle or the measures of its angles?

Activity

10.3

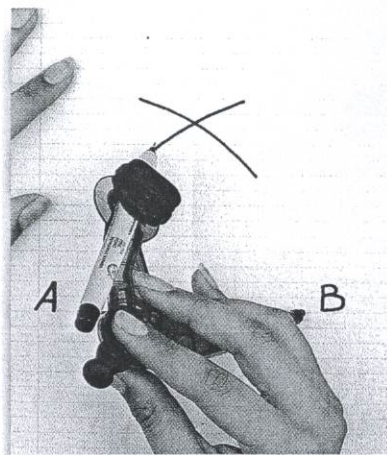
Special Right Triangles

Question

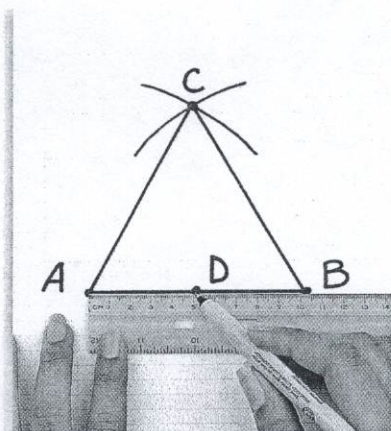
What is special about the ratios of the side lengths in a triangle with angle measures 30° , 60° , and 90° ?

Explore

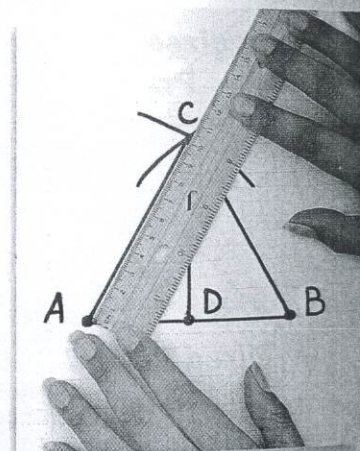
- 1 Draw a segment at least 10 centimeters long. Label it \overline{AB} . Set your compass opening to AB . Draw arcs with center A and center B .



- 2 Label the intersection of the arcs C . Draw equilateral $\triangle ABC$. Use your ruler to locate the midpoint of \overline{AB} . Label it D .



- 3 Draw \overline{CD} . $\triangle ACD$ has angle measures of 30° , 60° , and 90° . Measure AC , AD , and CD to the nearest millimeter.



Think About It

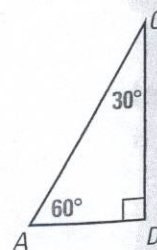
In Exercises 1–3, AC , AD , and CD are the lengths in centimeters of the sides of triangles similar to the one you constructed. Copy and complete the table. In Exercise 4, use the values from your drawing.

Student Help

LOOK BACK

To review ratios of side lengths in similar triangles, see p. 365.

	AC	AD	CD	$\frac{AC}{AD}$	$\frac{CD}{AD}$
1.	10	5	8.7	?	?
2.	20	10	17.3	?	?
3.	50	25	43.3	?	?
4.	?	?	?	?	?



5. What do you notice about the ratios $\frac{AC}{AD}$ and $\frac{CD}{AD}$ for $\triangle ACD$ with $m\angle A = 60^\circ$, $m\angle C = 30^\circ$, and $m\angle D = 90^\circ$?