**The History Of Trigonometry**

Trigonometry is often a source of difficulty and grief for students at both the high

school and collegiate level. The usefulness of trigonometry in the real world, in surveying, land measure, mensuration, and navigation, insured the importance of the subject in colonial times (Allen 71). High school students in modern times have their first experience in trigonometry in 10th or 11th grade using the concepts of sine, cosine, and tangent, to measure the angles of a right plane triangle in courses like pre-calculus or trigonometry. The emphasis in the study of plane trigonometry in the high school is changing from the study of measurement to the study of functions. The problem is that even when simply using these concepts to measure angles of a triangle, many students do not see the connections to real life, or understand where these concepts came from. Rather than incorporating trigonometric ideas with algebra and geometry, its predecessors, it is often introduced as a separate entity in the mathematical world of knowledge. As a result of having no connection to previously acquired mathematical learning, students are baffled by the intricacies of the subject and question the purpose it fulfills, not only in mathematics, but in everyday life as well. Perhaps by providing a brief insight into where these concepts came from, how they were discovered, and how their uses in the past relate

to how they are currently used and taught may provide students with the extra

understanding they need to put these concepts to use, whether it be as measuring devices, or as functions. With this better insight, one would be able to see the value in studying trigonometry as a component of mathematics, instead of a detached unit from the subject.

The term “trigonometry,” although not of native Greek origin, comes from the

Greek word *trigonon*, meaning “triangle,” and the Greek word *-metria*, meaning

“measurement.” As the name implies, trigonometry ultimately developed from the study of right triangles by applying the relationships between the measures of its sides and angles to the study of similar triangles (Gullberg 458). However, the word

“trigonometry” did not exist upon the birth of the subject, but was later introduced by the German mathematician and astronomer, Bartholomaeus Pitiscus in the title of his work, *Trigonometria sive de solutione triangularum tractatus brevis et perspicius…*, published in 1595. It was then revised in 1600 and published again as *Trigonometria sive de dimensione triangulae.*

As far as the origin of the subject is concerned, trigonometry and the development

of trigonometric functions have a rich, diverse history. Trigonometry is not the work of one man or a nation. In fact, the ancient Egyptians and Babylonians had developed theorems on ratios of the sides of similar triangles (Boyer 158), before trigonometry was ever formalized as a subdivision of mathematics. These two groups had no clear usage of trigonometric functions but were able to use them unknowingly to their advantage.

Egyptians used trigonometry to their benefit in land surveying and the building of

pyramids. Babylonian astronomers related trigonometric functions to arcs of circles and the lengths of chords subtending their arcs (Gullberg 458). The ancestral beginnings of trigonometry are thought to be the first numerical sequences correlating shadow lengths with the time of day. The shadow tables are the ancestors of cotangent and tangent; hence, these functions were later derived from these early discoveries. Shadow tables consisted of simple sequences of numbers, which primitively applied, “the fact that the shadow of a vertical stick (gnomon)—or of a person, for that matter—is long in the early 3 morning, shortens to a minimum at noon, then becomes longer and longer as the afternoon wears on” (Kennedy 335). The shadow tables related a particular hour of time with a particular length of the gnomon shadow, and were used as early as 1500 B.C. by the Egyptians. Similar tables were developed later by other civilizations, including the Indians and Greeks. The results of the shadow tables varied among the different civilizations, since shadow lengths were dependent on the position of the sun in relation to the place on Earth of each civilization that the shadow observations were taking place. It

is important to note that these civilizations were inadvertently corresponding shadow

lengths to time as a function; this attests that, at least 3000 years ago, humans inherently used the notion of a function, before even knowing or understanding what a function was.

In retrospect, shadow tables were a great development in the creation of trigonometry, but it was really the Greeks, however, who first developed trigonometry into an ordered science. “Trigonometry, perhaps more than any other branch of mathematics, developed as the result of a continual and fertile interplay of supply and demand: the supply of applicable mathematical theories and techniques available at any given time and the demands of a single applied science, astronomy” (Kennedy 333). The Greeks took over from the Babylonians as astronomers and studied the relationship between angles in a circle and lengths of chords to develop theories of planetary position and motion (Mankiewicz 18). By studying Babylonian planetary theory and astronomy, the Greeks inherited the sexagesimal number system, which is based on the number sixty (rather than our current decimal number system based on ten). “Although sixty may appear to be a large value to have as a base, it does convey certain advantages. Sixty is the smallest number that can be wholly divided by two, three, four, five, and six, and of course, it can also be divided by ten, fifteen, twenty, and thirty” (maxmon.com/1900bc.htm). Other than that, little is known and much is lost in prehistory. “Knowledge of the subject did not grow steadily. It progressed instead by a series of discontinuous jumps” (Kennedy 335). From the primitive landmarks of shadow tables and the Greeks’ gain and

expansion of astronomical knowledge from the Babylonians, there was a gap in the

improvement of trigonometry until the time of Hipparchus. The earliest contributions to sine, cosine and tangent came during the Alexandrian Period (300BC and 30BC).

Hipparchus, a highly credited Greek astronomer who came to be known as “the father of trigonometry,” had a great influence in the developments of trigonometry and is the first person whose use of trigonometry is documented (Heath 257). Trigonometric tables were created for computations related to the scientific field known as astronomy. The stars were thought to have been fixed on a sphere of great size. Only the planets moved on the sphere and therefore, to understand these positions on the sphere, mathematicians used spherical geometry (alepho.clarku.edu), which is the geometry of circles, angles, and

figures on the surface of a sphere. Hipparchus was the first person to create such a

trigonometric table of ratios. He did so by considering every triangle as being inscribed in a circle of large fixed radius.

Source: <http://www.math.rutgers.edu/~mjraman/History_Of_Trig.pdf>