

The Addition Property of Equality

Learning Objectives:

1. Identify linear equations.
2. Use the addition property of equality.
3. Simplify, and then use the addition property of equality.

Examples:

1. Decide whether each is an *expression* or an *equation*. If it is an *expression*, simplify it. If it is an *equation*, solve it.
 - a) $3x + 10 - x - 3$
 - b) $-7y + 13 + 8y = -6$
2. Which of the pairs of equations are equivalent equations?
 - a) $x + 5 = 8$ and $x = 3$
 - b) $-12 = x - 10$ and $x = -2$
 - c) $x - 15 = 5$ and $x = 6$
 - d) $x + 23 = 30$ and $x = -9$
3. Which of the following are linear equations in one variable?
 - a) $4x - 7 = 3x + 2$
 - b) $x^2 = 2x + 5$
 - c) $2x - 8 = 0$
 - d) $x^3 = 8x$
4. Solve for x . Check your answers.
 - a) $x - 4 = 16$
 - b) $14 = x - 12$
 - c) $x + 15 = 18$
 - d) $-19 = x + 16$
 - e) $21 = -16 + x$
 - f) $x - (-6) = 18$
 - g) $22 - 7 = x - 5$
 - h) $19 - 6 + x = 15 - 2$
 - i) $-26 + x = -42 + 10$
5. Solve for x . Check your answers.
 - a) $\frac{1}{4} + x = \frac{3}{4}$
 - b) $\frac{1}{3} + x = \frac{5}{6}$
 - c) $x - \frac{1}{5} = 0$
 - d) $x - \frac{9}{10} = -\frac{2}{3} + \frac{1}{15}$
 - e) $-2.2 + x = 16$
 - f) $6x = 19.2 + x - 3.2$

Teaching Notes:

- Encourage students to write all of the addition property steps and to avoid using shortcuts until they have mastered these types of equations.
- Encourage students to write the steps for solving the equations in a neat and organized manner. This habit will help immensely when the equations become more complex.
- Encourage students to simplify both sides of the equation before using the addition principle.
- Refer students to the addition property of equality definition in the text.

Answers: 1a) *expression*; $2x + 7$, b) *equation*; $\{-19\}$; 2) choices a and b; 3) choices a and c; 4a) $\{20\}$, b) $\{26\}$, c) $\{3\}$, d) $\{-35\}$, e) $\{37\}$, f) $\{12\}$, g) $\{20\}$, h) $\{0\}$, i) $\{-6\}$; 5a) $\{\frac{1}{2}\}$, b) $\{\frac{1}{2}\}$, c) $\{\frac{1}{5}\}$, d) $\{\frac{9}{30}\}$, e) $\{18.2\}$, f) $\{3.2\}$

The Multiplication Property of Equality

Learning Objectives:

1. Use the multiplication property of equality.
2. Simplify, and then use the multiplication property of equality.

Examples:

1. By what number is it necessary to multiply each side of each equation in order to isolate x on the left side? Do not solve.
 - a) $\frac{1}{5}x = 3$
 - b) $\frac{x}{6} = -2$
 - c) $-\frac{3}{4}x = 21$
 - d) $-x = 41$
2. Tell whether you would use the addition or multiplication property of equality to solve each equation. Do not solve.
 - a) $4x + 3 = 3x$
 - b) $5x = -25$
3. Solve for x . Be sure to reduce your answers. Check your answers.
 - a) $\frac{1}{5}x = 6$
 - b) $\frac{1}{4}x = -25$
 - c) $\frac{x}{12} = 5$
 - d) $-9 = \frac{x}{9}$
4. Solve for x . Be sure to reduce your answers. Check your answers.
 - a) $3x = 9$
 - b) $7x = -56$
 - c) $-11 = 2x$
 - d) $1.2x = 88$
 - e) $-16 = -x$
 - f) $-x = 100$
 - g) $-3.3x = -111$
 - h) $53 = -8x$
 - i) $8x + 4 - 4x = 6 + x$
 - j) $x - 7 = 3x - 7$
5. Determine whether the given solution is correct. If it is not, find the correct solution.
 - a) $-2x = 12$; $\{6\}$
 - b) $-x = 45$; $\{-45\}$
6. Mixed practice.
 - a) $\frac{1}{6}x = -9$
 - b) $-43 = 4x$
 - c) $-9.8x = -210.994$
 - d) $-99 = -x$
 - e) $-8 = \frac{x}{8}$
 - f) $\frac{x}{15} = 225$

Teaching Notes:

- Some students need to be shown that $\frac{1}{12}x = \frac{1}{12} \cdot \frac{x}{1} = \frac{1 \cdot x}{12 \cdot 1} = \frac{x}{12}$.
- Refer to the multiplication property of equality definition in the text.

Answers: 1a) 5, b) 6, c) $-\frac{4}{3}$, d) -1; 2a) addition property of equality, b) multiplication property of equality;

3a) $\{30\}$, b) $\{-100\}$, c) $\{60\}$, d) $\{-81\}$; 4a) $\{3\}$, b) $\{-8\}$, c) $\left\{-\frac{11}{2}\right\}$ or $\left\{-5\frac{1}{2}\right\}$, d) $\{73.\bar{3}\}$, e) $\{16\}$,

f) $\{-100\}$, g) $\{33.\bar{6}\}$, h) $\{-6.625\}$, i) $\left\{\frac{2}{3}\right\}$, j) $\{0\}$; 5a) no; $\{-6\}$, b) yes; 6a) $\{-54\}$, b) $\{-10.75\}$, c) $\{21.53\}$,

d) $\{99\}$, e) $\{-64\}$, f) $\{3375\}$

More on Solving Linear Equations

Learning Objectives:

1. Learn and use the four steps for solving a linear equation.
2. Solve equations with fractions or decimals as coefficients.
3. Solve equations that have no solutions or infinitely many solutions.
4. Write expressions for two related unknown quantities.

Examples:

1. Determine whether the given solution is correct.
 - a) Is $\{12\}$ a solution for $5x + 4 - 2x = 3x - 5 + x$?
 - b) Is $\{9\}$ a solution for $5x + 4 - 2x = 3x - 5 + x$?
2. Solve for x . Check your solution.

a) $10x + 7 = 107$	b) $7x - 8 = 27$	c) $33 = 6x - 3$
d) $164 = 15x + 14$	e) $\frac{1}{2}x - 8 = -2$	f) $-\frac{2}{3}x - 8 = -32$
3. Solve the equation. Check your solution.

a) $4x = -2x + 60$	b) $8x - 6 = 3 + 9x$	c) $-6x - 10 = -7 + 10x$
d) $0.6y - 0.3 = 0.7 - 0.3y$	e) $x - 12 = 10 - x$	f) $-9x + 4 + 7x = -3x + 9$
4. Solve the equation. Check your solution.

a) $6(2x - 1) = 30$	b) $-1(x + 11) = 20$	c) $6(x - 8) = 6x - 48$
d) $3x + 6(x + 9) = 9x - 15$	e) $0.4x - 0.2(3 - x) = 7.6$	f) $7x - 3(x - 8) = 3x + 24$
5. Write the answers to each problem in terms of the variable.
 - a) Two numbers have a sum of 11. One number is y . What expression represents the other number?
 - b) Monica is 18 years old. What expression represents her age d years from now?
 - b) Enrico has k nickels. Express the value of the nickels in cents.

Teaching Notes:

- Encourage students to check their solutions, as in example 1.
- In example 3, some students prefer to always end up with the variable on the left, while others prefer to always end up with a positive coefficient in front of the variable.
- Some students confuse the different properties and try to subtract the coefficient from the variable instead of multiplying it off.
- Some students do not collect the like terms before trying to solve 3f.

Answers: 1a) no, b) yes; 2a) $\{10\}$, b) $\{5\}$, c) $\{6\}$, d) $\{10\}$, e) $\{12\}$, f) $\{36\}$; 3a) $\{10\}$, b) $\{-9\}$, c) $\{-\frac{3}{16}\}$,
 d) $\{\frac{10}{9}\}$ or $\{1\frac{1}{9}\}$, e) $\{11\}$, f) $\{5\}$; 4a) $\{3\}$, b) $\{-31\}$, c) $\{\text{all real numbers}\}$, d) \emptyset , e) $\{\frac{41}{3}\}$ or $\{13\frac{2}{3}\}$, f) $\{0\}$;
 5a) $11 - y$, b) $18 + d$, c) $5k$

An Introduction to Applications of Linear Equations

Learning Objectives:

1. Learn the six steps for solving applied problems.
2. Solve problems involving unknown numbers.
3. Solve problems involving sums of quantities.
4. Solve problems involving supplementary and complementary angles.
5. Solve problems involving consecutive integers.

Examples:

1. Suppose a problem requires you to find the number of points scored on a quiz. Which one of the following would not be a reasonable choice?
a) 0 b) -3 c) 14 d) 100
2. **Unknown number problems:**
 - a) The product of 6, and a number increased by 7, is 72. What is the number?
 - b) If 5 is added to a number, and the sum is tripled, the result is 11 more than the number. Find the number.
 - c) What number minus 310 gives 822?
 - d) When six is subtracted from half of a number, the result is -18. What is the original number?
 - e) One number is 24 greater than another number. The sum of the two numbers is -72. Find each number.
3. **Sum of quantities problem:**

In 2008, Pierre's Pizza and Burger Palace together had revenue totaling \$470,000. If Burger Palace took in \$90,000 less than Pierre's Pizza, how much did each take in as revenue?
4. **Supplementary and complementary angle problems:**
 - a) Find the measure of an angle whose complement is two times its measure.
 - b) Find the measure of an angle such that the sum of the measures of its complement and its supplement is 142° .
5. **Consecutive integer problems:**
 - a) The sum of two consecutive integers is 59. Find the integers.
 - b) When the lesser of two consecutive odd integers is added to twice the greater, the result is 187. Find the integers.

Teaching Notes:

- Many students find these problems difficult at first.
- Refer students to the **six-step strategy for solving applied problems** in the text.
- Encourage students to check whether their final answers are reasonable.

Answers: 1) choice b; 2a) 5, b) -2, c) 1132, d) -24, e) -48 and -24; 3) Pierre's Pizza had revenue of \$280,000 and Burger Palace had revenue of \$190,000; 4a) 30° , b) 64° ; 5a) 29 and 30, b) 61 and 63

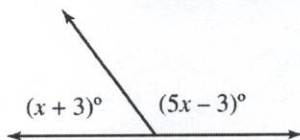
Formulas and Additional Applications from Geometry

Learning Objectives:

1. Solve a formula for one variable, given the values of the other variables.
2. Use a formula to solve an applied problem.
3. Solve problems involving vertical angles and straight angles.
4. Solve a formula for a specified variable.

Examples:

1. Decide whether *perimeter* or *area* would be used to solve a problem concerning the measure of the quantity.
 - a) Applying a weed and feed treatment for your lawn
 - b) Installing solar lights around the border of a swimming pool
2. Substitute values into the given formula and solve.
 - a) The formula for the perimeter of a rectangle is $P = 2L + 2W$. If the length, L , is 9 meters and the width, W , is 5 meters, find the perimeter, P , of the rectangle.
 - b) The area of a triangle is given by $A = \frac{1}{2}bh$. If the base, b , is 19 in. and the height, h , is 17 in., find the area.
3. The perimeter of a triangle is 37 feet. One side of the triangle is 2 feet longer than the second side. The third side is 5 feet longer than the second side. Find the length of each side.
4. Find the measure of each marked angle.



5. Solve for the indicated variable.

a) $d = rt$, for r	b) $V = \frac{1}{3}Bh$, for h	c) $P = 2L + 2W$, for W
d) $F = \frac{9}{5}C + 32$, for C	e) $y = 3x + 6$, for x	f) $S = 2\pi rh + 2\pi r^2$, for h

Teaching Notes:

- Most students like to enter all the given numbers into a formula before solving for one of the variables.
- Many students have difficulty with the problems in example 5.
- Sometimes it is required for students to solve a number of problems using the same formula. It may be advantageous for students to rewrite the formula so that it is solved for the required variable first.
- Encourage students to label answers with the correct units.

Answers: 1a) area, b) perimeter; 2a) $P = 28$ meters, b) 161.5 square inches; 3) 10 feet, 12 feet, and 15 feet;

4) $(x + 3)^\circ = 33^\circ$ and $(5x - 3)^\circ = 147^\circ$; 5a) $r = \frac{d}{t}$, b) $h = \frac{3V}{B}$, c) $W = \frac{P - 2L}{2}$, d) $C = \frac{5}{9}(F - 32)$, e) $x = \frac{y - 6}{3}$,

f) $h = \frac{S - 2\pi r^2}{2\pi r}$

Ratios, Proportions, and Percent

Learning Objectives:

1. Write ratios.
2. Solve proportions.
3. Solve applied problems using proportions.
4. Find percentages and percents.

Examples:

1. Write a ratio for each word phrase. Write fractions in lowest terms.
 - a) 30 lb to 50 lb
 - b) 125 dogs to 50 dogs
 - c) 20 min to 2 hr
 - d) 24 dollars to 48 dollars
2. Find the best buy (based on price per unit). Give the unit price to the nearest thousandth for that size.

Walnut Fudge	
Size	Price
5 lb	\$23.17
12 lb	\$54.38

3. Solve and check.
 - a) $\frac{2}{5} = \frac{x}{10}$
 - b) $\frac{x}{33} = \frac{9}{11}$
 - c) $\frac{2}{7} = \frac{7}{x}$
 - d) $\frac{1}{x} = \frac{6}{56}$
 - e) $\frac{12}{x+3} = \frac{6}{x}$
4. Solve and check.
 - a) On a map of Great Brook Hiking Trails, 1 centimeter corresponds to 4 miles. Find the length of a trail represented by a line that is $2\frac{1}{2}$ centimeters long on the map.
 - b) A quality control inspector found 4 defective computer chips in a shipment of 500 computer chips. At this rate, how many computer chips would be defective in a shipment of 2000 computer chips?
 - c) A caterer estimates that 6 gallons of punch will serve 40 people. How much *additional* punch would be required to serve 60 people?
5. Find the missing number or the missing percent.
 - a) What is 10% of 70?
 - b) What is 32% of 224?
 - c) Find 190% of 375.
 - d) 25 is what percent of 125?
 - e) What percent of 80 is 0.8?
 - f) 126 is what percent of 28?
6. Application problems.
 - a) The Smith family paid 22% of the purchase price of a \$231,000 home as a down payment. Determine the amount of the down payment.
 - b) A salesperson earned a commission of \$5316 for selling \$44,300 worth of batteries to various stores. Find the commission rate.

Teaching Notes:

- Many students find this section difficult.
- One way to remember which way to move the decimal point is to remember that the decimal is moved right for Decimal to Percent, and left for percent to decimal.

Answers: 1a) $\frac{3}{5}$, b) $\frac{5}{2}$, c) $\frac{1}{6}$, d) $\frac{1}{2}$; 2) 12 lb is cheaper, \$4.532/lb; 3a) $x = 4$, b) $x = 27$, c) $x = 49/2$, d) $x = 28/3$, e) $x = 3$; 4a) 10 miles, b) 16 computer chips, c) 3 extra gallons; 5a) 7, b) 71.68, c) 712.5, d) 20%, e) 1%, f) 450%; 6a) \$50,820, b) 12%

Solving Linear Equations

Learning Objectives:

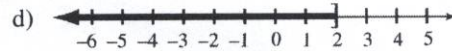
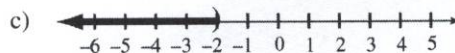
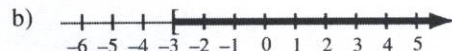
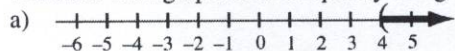
1. Graph intervals on a number line.
2. Use the addition property of inequality.
3. Use the multiplication property of inequality.
4. Solve inequalities using both properties of inequality.
5. Use inequalities to solve applied problems.
6. Solve linear inequalities with three parts.

Examples:

1. Graph each inequality on a number line.

a) $x > 1$ b) $x \geq -4$ c) $x < \frac{2}{3}$ d) $x \leq -2.5$

2. Translate each graph to an inequality using the variable x .



3. Solve each inequality and write the answer in interval notation.

a) $4x - 10 > 5x + 4$ b) $5x - 2 \leq 6x + 4$
 c) $-4x - 7 < 5(2x + 7)$ d) $7(x - 3) > 2(5x - 8)$
 e) $1 > 3x - 2 > -5$ f) $1 \leq 2x - 5 < 5$

4. Solve using an inequality.

- a) A certain car has a weight limit for all passengers and cargo of 1129 pounds. The four passengers in the car weigh an average of 165 pounds. Find the weight of the cargo that the car can handle.
- b) A certain store has a fax machine available for use by its customers. The store charges \$1.85 to send the first page and \$0.45 for each subsequent page. Find the number of pages that can be faxed for \$7.25.
- c) An archery set containing a bow and three arrows costs \$68. Additional arrows can be purchased for \$9 each. Jerry has \$230 to spend on the set and additional arrows. Including the arrows in the set, what is the total number of arrows Jerry can purchase?

Teaching Notes:

- Some students are unfamiliar with $<$ and $>$ and need to be told to point to the lesser number.
- Some students need to see examples in problem 2 using a number line.
- Some students benefit from seeing fraction examples for problems 1a and 1d.
- Review the common English to inequality translations.

Answers: 1a - d) see graph answer pages; 2a) $x > 4$, b) $x \geq -3$, c) $x < -2$, d) $x \leq 2$; 3a) $(-\infty, -14)$, b) $[-6, \infty)$,

c) $(-3, \infty)$, d) $(-\infty, -\frac{5}{3})$, e) $(-1, 1)$, f) $[3, 5)$; 4a) $x \leq 469$ pounds, b) $x \leq 12$ pages, c) $x \leq 21$ arrows

Solving Linear Equations

Learning Objectives:

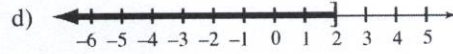
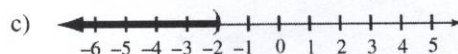
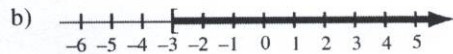
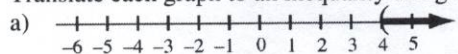
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