

AB

(7, 13)

Reflect on This

L'Ascension  
L. de la Trinite  
L. de Martin Vaz

$$y = ax_3$$



1  
module

# Flashbacks

NAME: \_\_\_\_\_

DATE: \_\_\_\_\_

## ACTIVITY

### 1

1.1 Make a sketch and write a short definition for each of the following geometric figures:

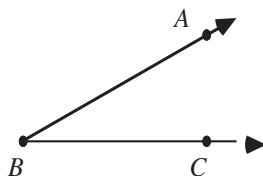
- a. an isosceles triangle      b. a square      c. a pentagon      d. an octagon.

1.2 Draw an angle with each of the following measures:

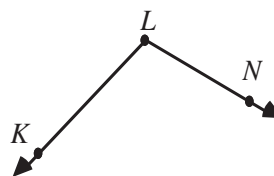
- a.  $35^\circ$       b.  $90^\circ$       c.  $120^\circ$

1.3 Estimate the measures of the two angles below. Use a protractor to check your estimates.

a.



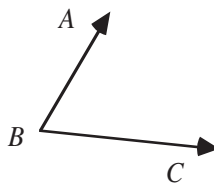
b.



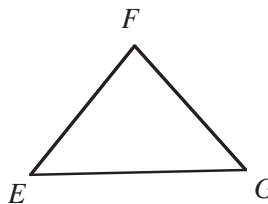
1.4 a. Create and name a segment that is congruent to the segment below:



b. Create and name an angle that is congruent to the angle below:



c. Create and name a figure that is congruent to the figure below:



1.5 Find the measure of  $\angle C$  in  $\triangle ABC$  given the measures of  $\angle A$  and  $\angle B$  below.

- a.  $m\angle A = 56^\circ$ ,  $m\angle B = 41^\circ$       b.  $m\angle A = 120^\circ$ ,  $m\angle B = 30^\circ$   
c.  $m\angle A = 40^\circ$ ,  $m\angle B = 40^\circ$

# Flashbacks

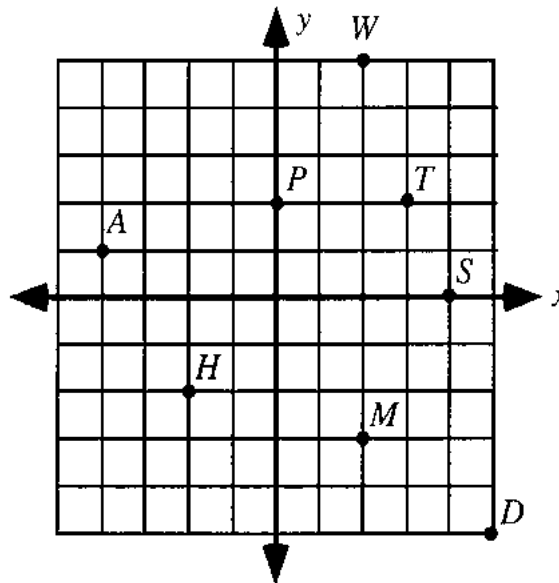
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## ACTIVITY

### 2

- 2.1 Draw and label an  $x$ -axis and a  $y$ -axis on a sheet of graph paper.
- Plot and label the following ordered pairs on your Cartesian coordinate system:  $A(0,0)$ ,  $B(5,3)$ ,  $C(-7,2)$ ,  $D(4,-4)$ , and  $E(-1,6)$ .
  - Find a point that is collinear to each of the following pairs of points:
    - $(2,4)$  and  $(2,-1)$
    - $(-4,5)$  and  $(3,5)$
    - $(3,4)$  and  $(-2,-1)$ .
- 2.2 Find the coordinates of each point on the following graph if each grid mark represents 1 unit.



- 2.3
- Use a protractor and straightedge to draw a pair of complementary angles.
  - Use a protractor and straightedge to draw a pair of supplementary angles.

# Flashbacks

NAME: \_\_\_\_\_

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## ACTIVITY

3

- 3.1 a. Draw a segment with endpoints  $(8,5)$  and  $(8,-4)$  on a sheet of graph paper.
- b. Draw a segment with one endpoint at  $(6,3)$  that intersects the  $x$ -axis and is parallel and congruent to the segment in Part a.
- c. Write the coordinates of the points where these two segments intersect the  $x$ -axis.
- 3.2 a. Draw a segment with endpoints  $(2,-10)$  and  $(-4,-10)$ .
- b. Draw a segment with one endpoint at  $(-2,-5)$  that intersects the segment in Part a and is perpendicular and congruent to it.

# Flashbacks

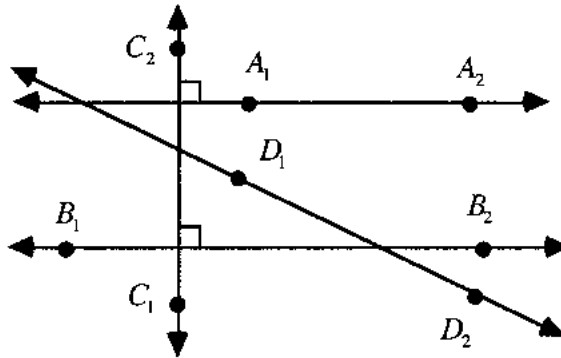
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## ACTIVITY

### 4

- 4.1 What is the measure of the central angle for a regular polygon with 14 sides?
- 4.2 Use the diagram below to complete Parts **a** and **b**.



- a. Name a pair of perpendicular line segments in the diagram.
- b. Name a pair of parallel line segments in the diagram.
- 4.3 Determine the coordinates of the image when the point  $A(3,5)$  is reflected:
- a. in the  $x$ -axis
- b. in the  $y$ -axis
- c. in the  $x$ -axis, then in the  $y$ -axis.

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## *Periodic Assessment 1—for use after Activity 2*

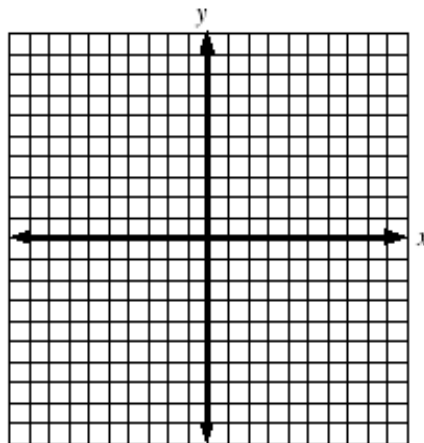
1. Determine the measure of the central angle for a regular polygon with:
  - a. 3 sides
  - b. 7 sides
  - c.  $p$  sides
2. Find the number of sides in a regular polygon whose central angles measure:
  - a.  $45^\circ$
  - b.  $51\frac{3}{7}^\circ$
  - c.  $m^\circ$ , where  $m < 360$
3. Identify each set of angle measures below as supplementary, complementary, or neither. Justify your answers.
  - a.  $52^\circ$ ,  $38^\circ$
  - b.  $30^\circ$ ,  $24^\circ$ ,  $36^\circ$
  - c.  $66^\circ$ ,  $114^\circ$
4. Determine the supplement and complement of each angle:
  - a.  $67^\circ$
  - b.  $112^\circ$
  - c.  $v^\circ$ , where  $v < 90$
5. The sum of the measures of the interior angles of a triangle is  $180^\circ$ . Is it possible to have a pair of supplementary angles in one triangle? Explain your answer.

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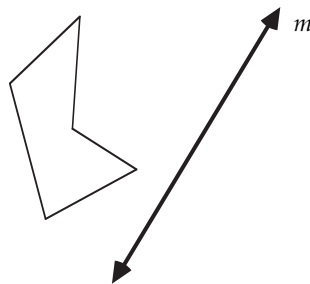
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**Periodic Assessment 2—for use after Activity 4**

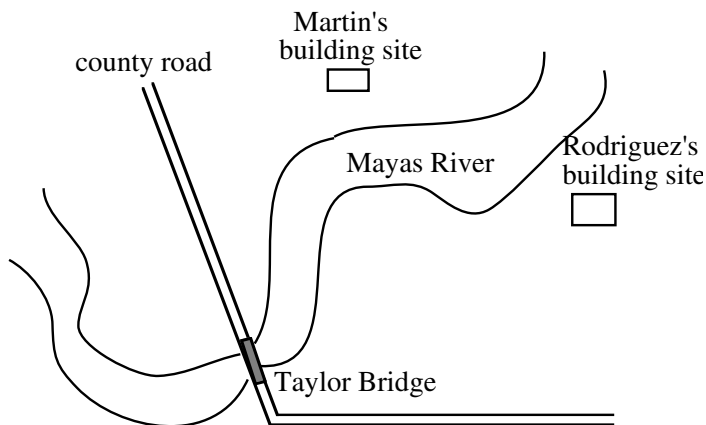
1. Plot the points  $A(-5,9)$  and  $B(6,-2)$  on the coordinate grid below. Plot the reflection of point  $A$  in the  $y$ -axis and the reflection of point  $B$  in the  $x$ -axis. Label the points and their reflections. Identify the coordinates of the images.



2. Sketch the reflection in the line  $m$  of the figure below.



3. The Martin and Rodriguez families are planning to build their new homes on adjacent parcels. Each family must build a driveway from their house to the county road. Because they are good friends, they want the driving distance between their homes to be as short as possible. To keep costs down, however, they do not want to build any new bridges. Where would you recommend that they locate the driveways? Justify your recommendation.

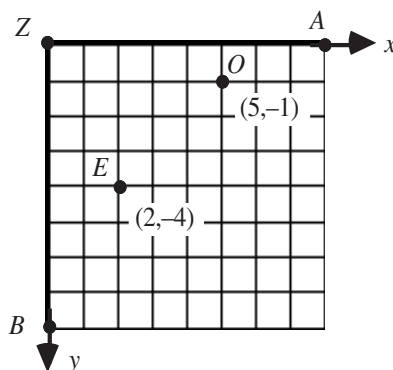


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## Module Assessment

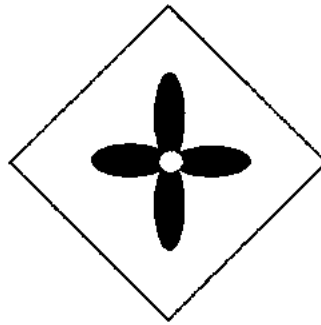
1.
  - a. Using a protractor and straightedge, draw a diagram of a light ray reflecting off a flat surface with an incoming angle of  $20^\circ$ .
  - b. Draw and label a normal to the surface.
  - c. Label and measure all the angles in the diagram.
2. Given points  $A(3, -2)$  and  $B(-2, 1)$ , determine the coordinates of  $A'$  and  $B'$  for each of the following:
  - a. a reflection in the  $x$ -axis
  - b. a reflection in the  $y$ -axis.
3. Mr. Ned is an intelligent, but lazy, horse. From his place in the pasture, he wants to walk to the stream for a drink, then back to the barn for a nap.
  - a. Draw a model that shows the shortest route Mr. Ned could take.
  - b. Explain why the route you identified in Part **a** is the shortest possible.
  - c. Identify a segment in your model from Part **a** that is a perpendicular bisector and describe its geometric features.
4. In the following diagram, point  $O$  represents an object and point  $E$  represents the eye. **Note:** The intersection of the  $x$ -axis and the  $y$ -axis of a Cartesian coordinate system always forms an angle of  $90^\circ$ .



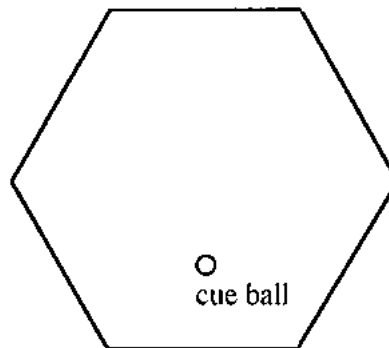
- a. If mirrors were placed on both axes in the diagram, determine the path a light ray would travel from  $O$  to  $E$  in a double reflection.
- b. Label the points of reflection on the  $x$ - and  $y$ -axes  $C_1$  and  $C_2$ , respectively.
- c. Label the two incoming and two outgoing angles and give their measures.
- d. Identify all the congruent angles and all the complementary angles.
- e. Describe the relationship between the path of the light ray from  $O$  to  $C_1$  and the path of the light ray from  $C_2$  to  $E$ .



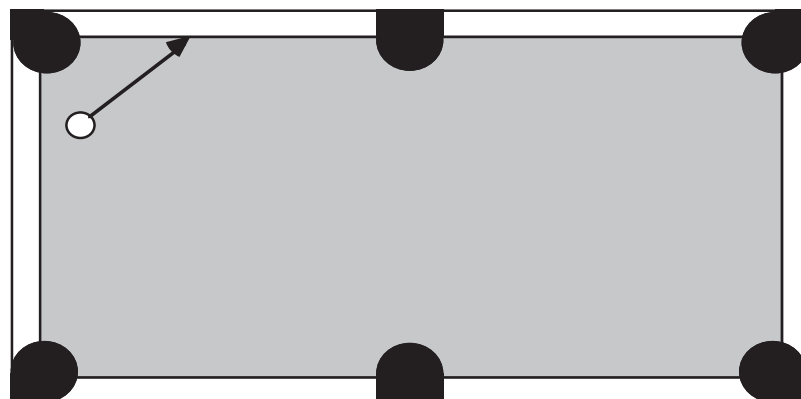
5. Find the measure of the central angle for each of the following:
  - a. a regular pentagon
  - b. a regular decagon
  - c. a regular  $n$ -gon.
6. Describe how the reflections in two hinged mirrors could be used to create the pattern below.



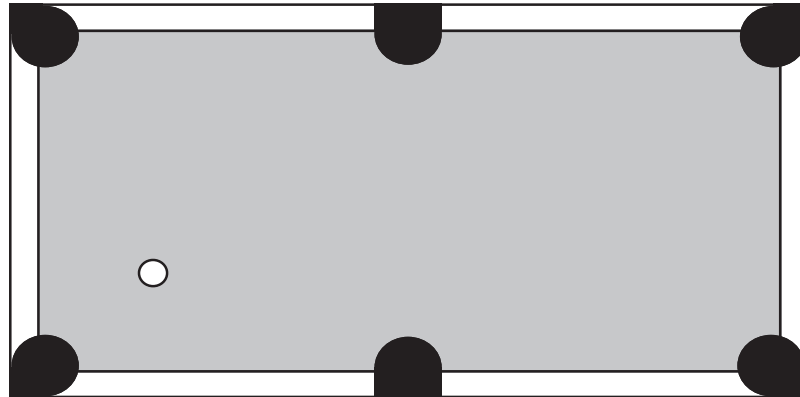
7. The diagram below shows a pool table shaped like a regular hexagon. Draw the path of a shot in which the ball bounces off exactly two side rails before returning to its original position.



8. The following diagram shows the path of a pool ball hit toward the side rail. If the ball is hit with enough force, will it eventually roll into one of the pockets? Justify your response, including a drawing.



9. The diagram below shows the location of a ball on a pool table. On a copy of this diagram, sketch the path of the ball that bounces off two side rails before falling into the upper right-hand corner pocket.

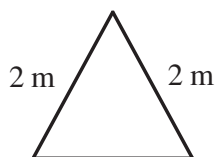


# Solutions to Flashbacks

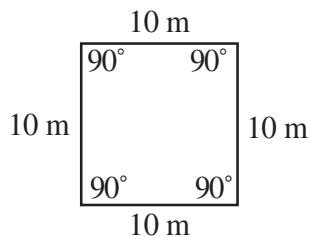
## ACTIVITY

### 1

- 1.1 a. An isosceles triangle is a triangle with at least two sides of equal length. The angle between the two sides of equal length is the vertex angle, while the other two angles are the base angles.



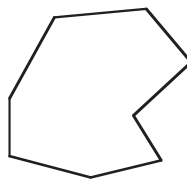
- b. A square is a quadrilateral with all four sides congruent and four right angles.



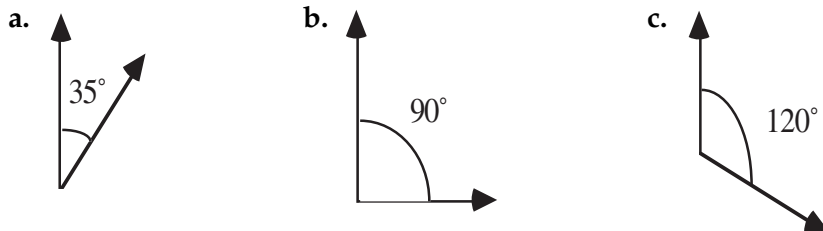
- c. A pentagon is a closed geometric figure with five sides.



- d. An octagon is a closed geometric figure with eight sides.



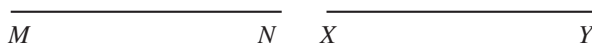
1.2 Sample response:



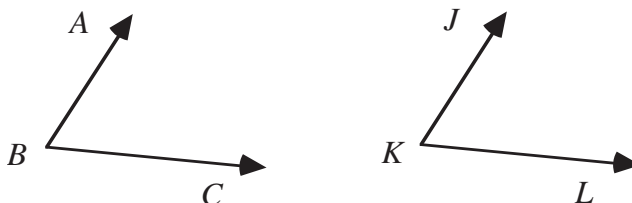
1.3 a.  $m\angle ABC = 29^\circ$

b.  $m\angle NLK = 106^\circ$

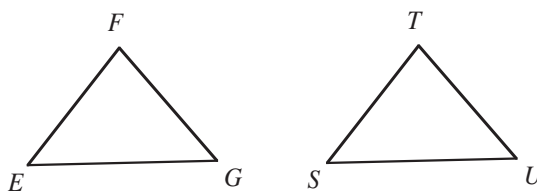
1.4 a. Any segment that is the same length as  $\overline{MN}$  is a congruent segment. Sample response:



b. Any angle that has the same degree measure as  $\angle ABC$  is a congruent angle. Sample response:



c. A triangle is congruent to  $\triangle EFG$  if corresponding angles are congruent and corresponding sides are congruent. Sample response:



1.5 a.  $m\angle C = 83^\circ$

b.  $m\angle C = 30^\circ$

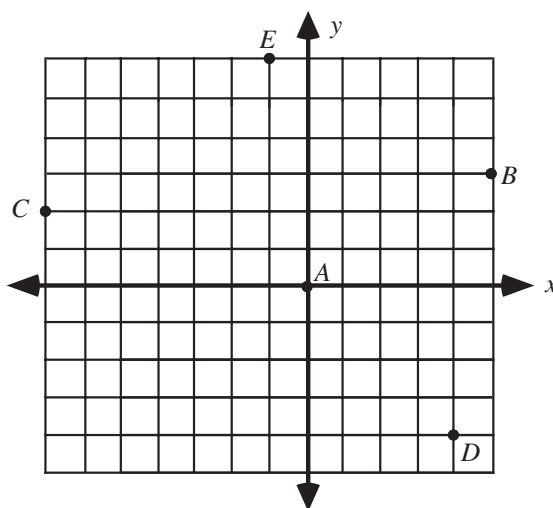
c.  $m\angle C = 100^\circ$

# Solutions to Flashbacks

## ACTIVITY

### 2

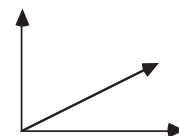
2.1 a. In the following sample graph, each grid mark represents 1 unit.



- b. 1. In general, any ordered pair  $(2,y)$  is collinear to  $(2,4)$  and  $(2,-1)$ .
2. In general, any ordered pair  $(x,5)$  is collinear to  $(-4,5)$  and  $(3,5)$ .
3. In general, any ordered pair  $(x,x+1)$  is collinear to  $(3,4)$  and  $(-2,-1)$ .

2.2 The ordered pairs are  $A(-4,1)$ ,  $D(5,-5)$ ,  $H(-2,-2)$ ,  $M(2,-3)$ ,  $P(0,2)$ ,  $S(4,0)$ ,  $T(3,2)$ , and  $W(2,5)$ .

- 2.3 a. The sum of the measures of the two angles must equal  $90^\circ$ . When placed adjacent to each other, with a common side, the two angles must form a right angle. A sample response is shown on the right.



- b. The sum of the measures of the two angles must equal  $180^\circ$ . When placed adjacent to each other, with a common side, the two angles must form a straight line. A sample response is shown below.

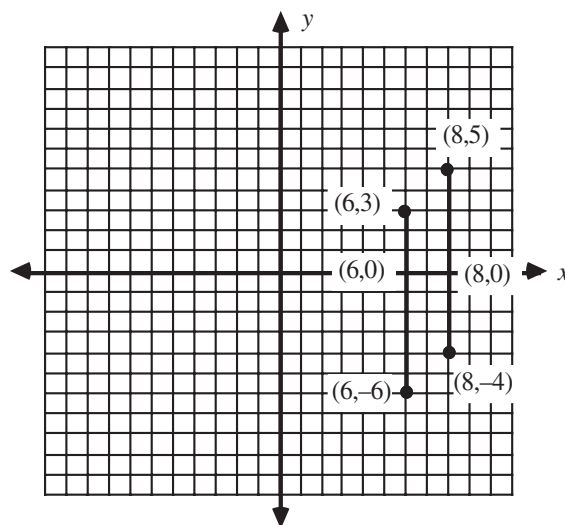


# Solutions to Flashbacks

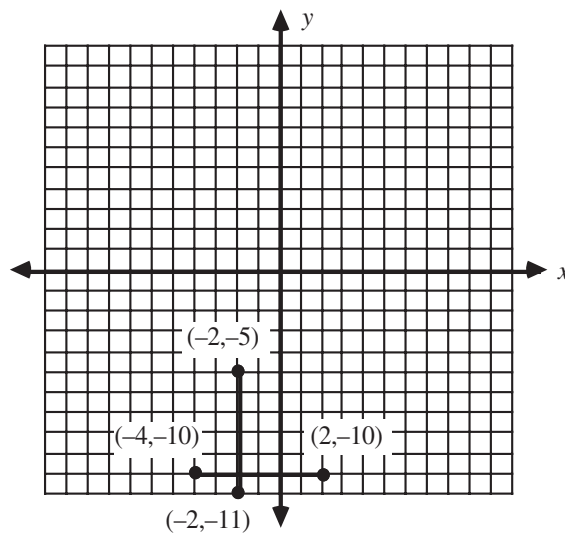
## ACTIVITY

### 3

3.1 a–c. In the following graph, each grid mark represents 1 unit.



3.2 a–b. In the following graph, each grid mark represents 1 unit.



# *Solutions to Flashbacks*

## ACTIVITY

### 4

- 4.1 The measure of the central angle for a regular polygon with 14 sides is  $25\frac{5}{7}^\circ \approx 25.71^\circ$ .
- 4.2 a. One pair of perpendicular line segments is  $\overline{C_1C_2}$  and  $\overline{B_1B_2}$ .  
b. One pair of parallel line segments is  $\overline{A_1A_2}$  and  $\overline{B_1B_2}$ .
- 4.3 a.  $(3, -5)$   
b.  $(-3, 5)$   
c.  $(-3, -5)$

## Solutions to Periodic Assessment 1

1.
  - a.  $120^\circ$
  - b.  $51\frac{3}{7}^\circ$
  - c.  $(360/p)^\circ$
2.
  - a. 8 sides
  - b. 7 sides
  - c.  $360/m$  sides
3.
  - a. Complementary. The sum of the two angle measures is  $90^\circ$ .
  - b. Neither. The terms *supplementary* and *complementary* apply to pairs of angles, not three angles.
  - c. Supplementary. The sum of the two angle measures is  $180^\circ$ .
4.
  - a. supplement =  $113^\circ$ ; complement =  $23^\circ$
  - b. supplement =  $68^\circ$  (there is no complement)
  - c. supplement =  $(180 - v)^\circ$ ; complement =  $(90 - v)^\circ$
5. Sample response: No. Two supplementary angles have a sum of  $180^\circ$ . The sum of the interior angles of a triangle is  $180^\circ$ . It would be impossible to have a third angle and keep the sum at  $180^\circ$ .

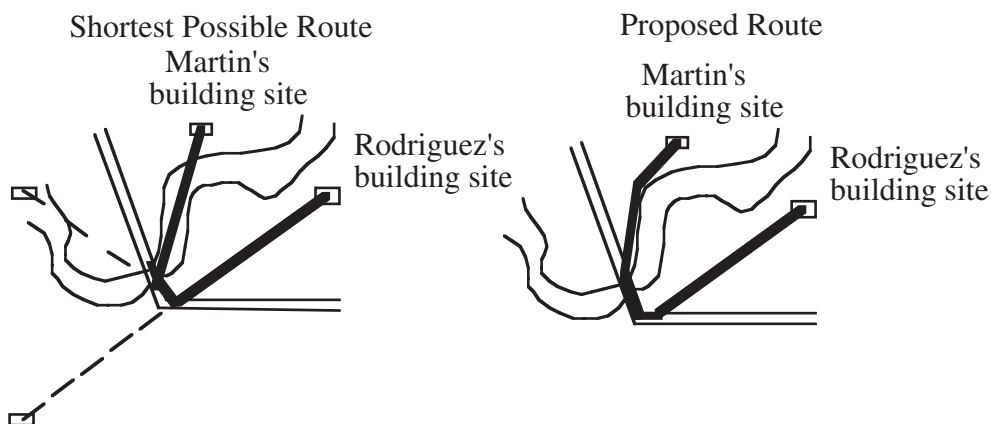


## Solutions to Periodic Assessment 2

1.  $A' = (5,9)$ ,  $B' = (6,2)$
2. Student sketches should resemble the one below.

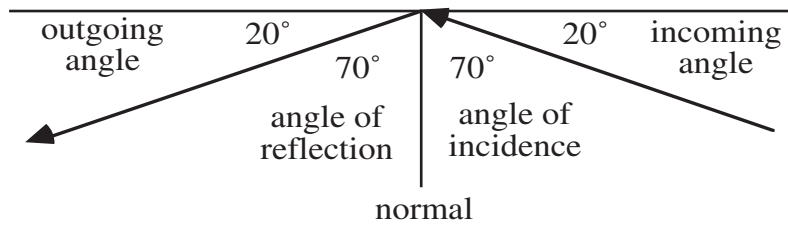


3. Sample response: The shortest possible route can be found by using a double reflection of the Martin family's building site in the two segments of the county road. Because this route crosses the river and does not use the existing bridge, it must be adjusted. The proposed route shown below uses the existing bridge. This will keep costs down and create a reasonably short route.



## Solutions to Module Assessment

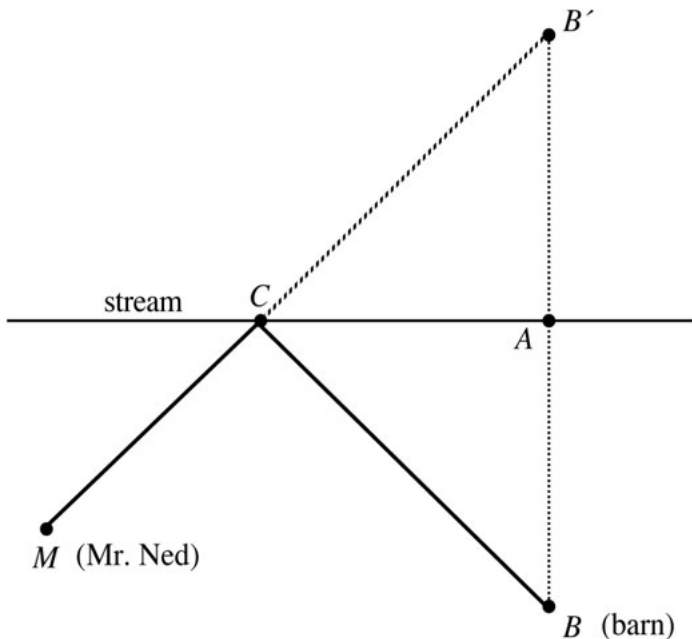
1. a–c. Sample response:



2. a.  $A'(3,2)$  and  $B'(-2,-1)$

b.  $A'(-3,-2)$  and  $B'(2,1)$

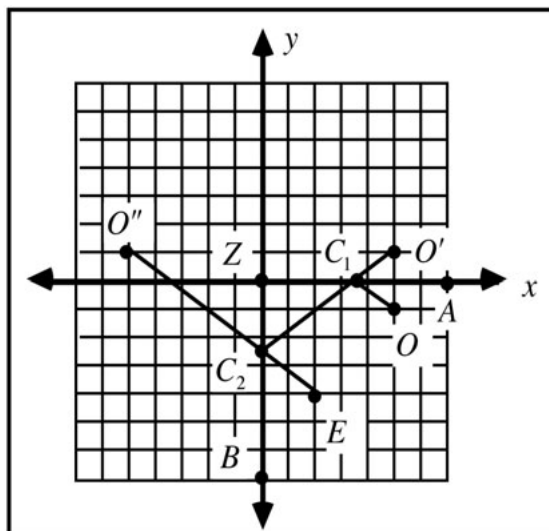
3. a. Sample response:



b. Sample response: In this case, the stream is the mirror line,  $B'$  is the image of the barn reflected in the stream, and  $C$  is the reflection point. The path with the shortest total distance always passes through the reflection point on the mirror line.

c. In the sample diagram above, the segment that represents the stream is a perpendicular bisector of segment  $\overline{BB'}$ . The perpendicular bisector divides  $\overline{BB'}$  into two congruent segments,  $\overline{BA}$  and  $\overline{AB'}$ . It also intersects  $\overline{BB'}$  at a  $90^\circ$  angle. Therefore,  $\angle CAB$  and  $\angle CAB'$  both measure  $90^\circ$ .

4. a–b. Sample response:



c. The two incoming angles are  $\angle AC_1O$  and  $\angle C_1C_2Z$ . Their measures are  $36^\circ$  and  $54^\circ$ , respectively. The two outgoing angles are  $\angle C_2C_1Z$  and  $\angle BC_2E$ . Their measures are also  $36^\circ$  and  $54^\circ$ , respectively.

d. Because an incoming angle is congruent to its outgoing angle,  $\angle AC_1O$  is congruent to  $\angle C_2C_1Z$ , and  $\angle BC_2E$  is congruent to  $\angle C_1C_2Z$ .

Because  $m\angle C_2ZC_1$  is  $90^\circ$  and the sum of the angles of a triangle is  $180^\circ$ ,  $\angle C_2C_1Z$  and  $\angle C_1C_2Z$  are complementary. Because  $\angle C_2C_1Z$  is congruent to  $\angle AC_1O$ ,  $\angle AC_1O$  and  $\angle C_1C_2Z$  are also complementary. Similarly,  $\angle C_2C_1Z$  and  $\angle BC_2E$  are complementary, as are  $\angle AC_1O$  and  $\angle BC_2E$ .

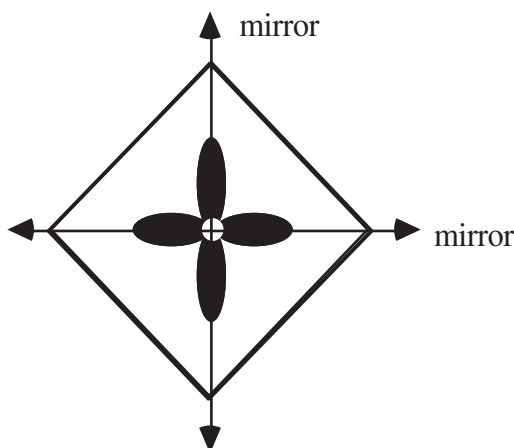
e. The path of the light ray from  $O$  to  $C_1$  is parallel to the path of the light ray from  $C_2$  to  $E$ , but their directions are opposite.

5. a.  $72^\circ$

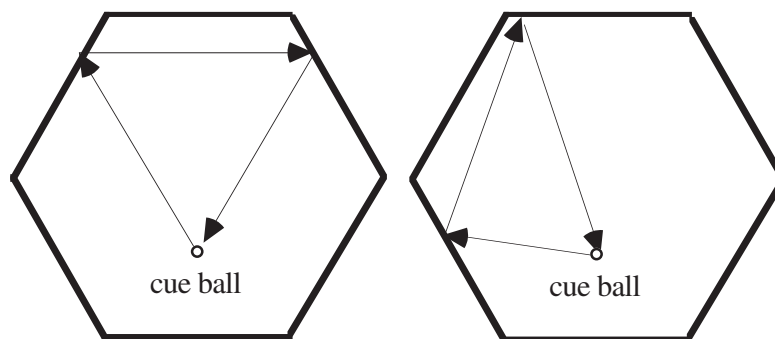
b.  $36^\circ$

c.  $360^\circ/n$

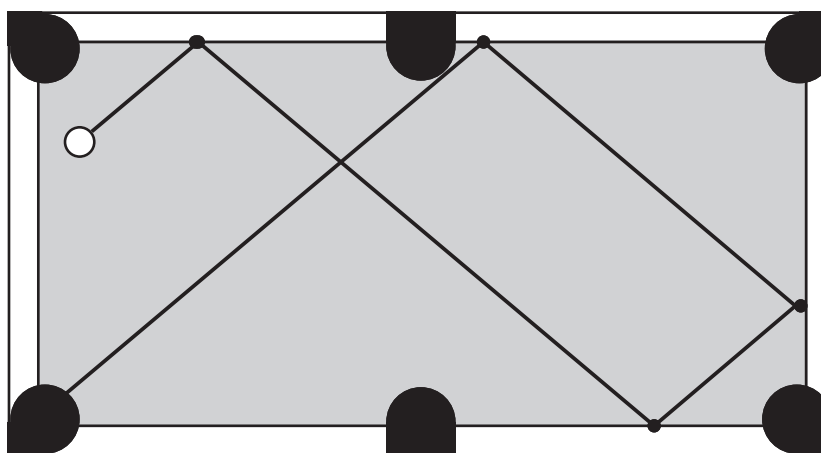
6. As shown in the diagram below, the two mirrors should be placed along two perpendicular axes, with their intersection at the center of the pattern:



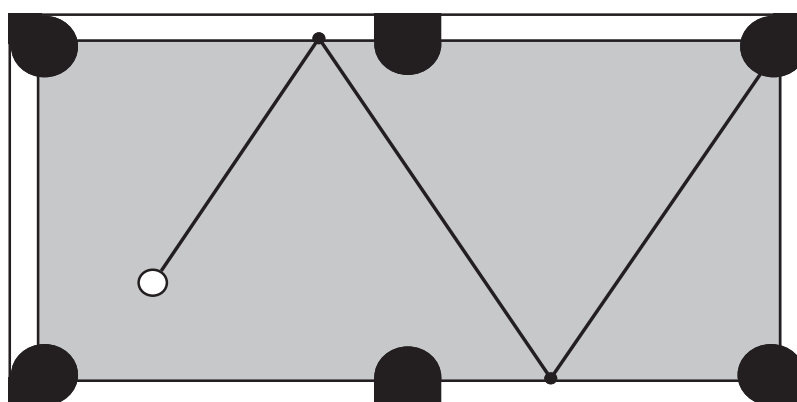
7. Answers may vary. The diagram below shows two possible paths.



8. The ball will fall in the lower left-hand corner pocket after four banks, as shown in the diagram below.



9. One possible double-bank shot is shown below.



## Template for Cartesian Coordinate System

