



Reteaching

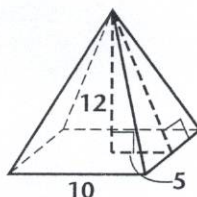
7.3 Surface Area and Volume of Pyramids

◆ Skill A Finding the surface area of a regular pyramid

Recall The surface area, S , of a regular pyramid with lateral area L and base area B is $S = L + B$. If the slant height of the pyramid is ℓ and the perimeter is p , the surface area is $S = \frac{1}{2}\ell p + B$. Also, $B = \frac{1}{2}ap$, where a is the apothem.

◆ Example

Find the surface area of the regular square pyramid.



◆ Solution

The base is a square with sides that are 10 units long, so $B = 100$.

The perimeter of the base is 40 units.

Next use the Pythagorean Theorem to find the slant height, ℓ .

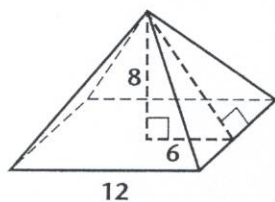
$$\begin{aligned}\ell^2 &= 5^2 + 12^2 \\ &= 25 + 144 \\ &= 169 \\ \ell &= 13\end{aligned}$$

$$\text{Then } S = \frac{1}{2}\ell p + B = \frac{1}{2}(13)(40) + 100 = 360.$$

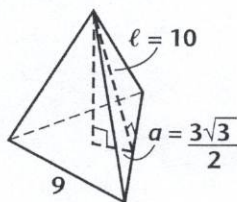
The surface area of the pyramid is 360 square units.

Find the surface area of each regular pyramid.

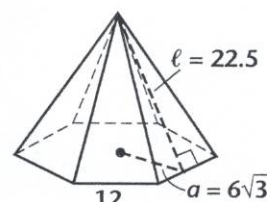
1.



2.



3.



Find the surface area of each pyramid.

4. a regular pyramid whose base is a pentagon with sides 4 inches long and area 43 square inches, and slant height 10 inches _____
5. a regular pyramid whose base is a hexagon with sides 9 meters long and apothem 7.8 meters, and slant height 12 meters _____

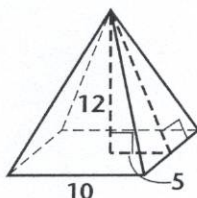
6. a regular pyramid whose base is an equilateral triangle with sides 7 feet long
and apothem 2 feet, and slant height 4 feet _____

◆ **Skill B** Finding the volume of a pyramid

Recall The volume, V , of a pyramid with base area B and height h is $V = \frac{1}{3}Bh$.

◆ **Example**

Find the volume of a regular square pyramid.



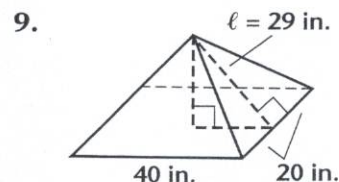
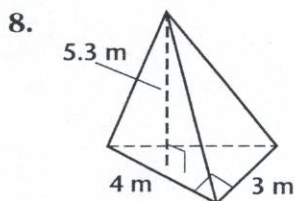
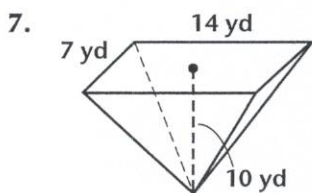
◆ **Solution**

The area of the base is 100 square units and the height is 12 units.

$$\begin{aligned} V &= \frac{1}{3}Bh \\ &= \frac{1}{3}(100)(12) \\ &= 400 \end{aligned}$$

The volume is 400 cubic units.

Find the volume of each pyramid.



10. an octagonal pyramid with base area 412 square meters and height 15 meters _____
11. a pentagonal pyramid with base area 108 square inches and height 7.5 inches _____
12. a rectangular pyramid whose base is 20 centimeters long and 16 centimeters
wide, and whose height is 10 centimeters _____
13. a pyramid whose base is a right triangle with sides of length 9 feet, 12 feet, and 15
feet and whose height is 10 feet _____



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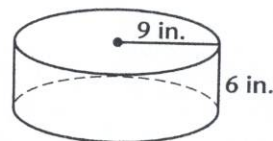
7.4 Surface Area and Volume of Cylinders

◆ Skill A Finding the surface area of a right cylinder

Recall The surface area, S , of a cylinder is the sum of its lateral area and the area of both bases, that is, $S = L + 2B$. For a right cylinder, if the radius of a base is r and the height of the cylinder is h , the surface area, S , is $S = 2\pi rh + 2\pi r^2$.

◆ Example 1

The radius of a base of a right cylinder is 9 inches. The cylinder is 6 inches high. Find the exact surface area of the cylinder and the surface area rounded to the nearest tenth.



◆ Solution

$r = 9$ and $h = 6$, so $S = 2\pi(9)(6) + 2\pi(9^2) = 270\pi$.

The surface area is 270π square inches, or approximately 848.23 square inches.

◆ Example 2

A right cylinder that is 15 meters high has surface area 1400π square meters. Find the exact circumference of a base of the cylinder.

◆ Solution

To find the circumference, first find the radius. $S = 2\pi rh + 2\pi r^2$

$$1400\pi = 2\pi r(15) + 2\pi r^2$$

$$700 = 15r + r^2$$

Solve the quadratic equation: $r^2 + 15r - 700 = 0$; $r = -35$ or $r = 20$

Since r must be positive, $r = 20$. Then $C = 2\pi r = 40\pi$.

The circumference of a base is 40π meters.

The dimensions of a right cylinder are given. Find the exact surface area of the cylinder and the surface area rounded to the nearest hundredth.

1. radius of a base = 12 in., height = 12 in.

2. radius of a base = 2 ft, height = 1 ft

3. radius of a base = 30 cm, height = 8 cm

4. diameter of a base = 1 mm, height = 1 mm

Find the unknown value for each cylinder. Give the exact value as well as the value rounded to the nearest tenth.

5. surface area = $92\pi \text{ m}^2$, radius of a base = 5 m, height = _____

6. surface area = $507\pi \text{ ft}^2$, height = 12 ft, diameter of a base = _____

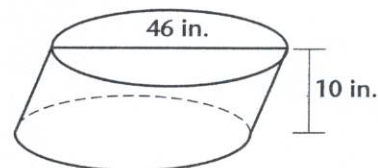
7. surface area = $7500\pi \text{ yd}^2$, height = 12 yd, circumference of a base = _____

◆ Skill B Finding the volume of a cylinder

Recall If the area of a base of a cylinder is B and the height of the cylinder is h , then the volume, V , of the cylinder is $V = Bh$. If the radius of the cylinder is r , then the volume is $V = \pi r^2 h$.

◆ Example

The diameter of a base of an oblique cylinder is 46 inches. The height of the cylinder is 10 inches. Find the exact volume of the cylinder and the volume rounded to the nearest tenth.

**◆ Solution**

$$V = \pi r^2 h$$

Since $d = 46$, $r = 23$.

Then, since $h = 10$,

$$\begin{aligned} V &= \pi(23)^2(10) \\ &= 5290\pi. \end{aligned}$$

The volume of the cylinder is 5290π cubic units, or approximately 16,619.0 cubic inches.

Find the exact volume of each cylinder. Then give the volume rounded to the nearest tenth.

8. area of a base = 68 cm^2 , height = 12 cm

9. radius of a base = 1 in., height = 4 in.

10. diameter of a base = 20 m, height = 8 m

11. radius of a base = 2.4 cm, height = 10 cm

12. radius of a base = 5 ft, height = 4 ft

13. circumference of a base = 24π , height = 6

Find the unknown value for each cylinder. Give the exact value as well as the value rounded to the nearest tenth. In Exercises 16–17, the cylinder is a right cylinder.

14. volume = $128\pi \text{ cm}^3$, diameter of a base = 8 m, height = _____

15. volume = $245\pi \text{ in.}^3$, height = 5 in., circumference of a base = _____

16. surface area = $80\pi \text{ m}^2$, height = 3 m, volume = _____

17. surface area = $250\pi \text{ m}^2$, height = 20 m, volume = _____



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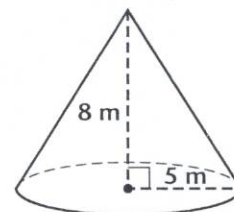
7.5 Surface Area and Volume of Cones

◆ Skill A Finding the surface area of a right cone

Recall The surface area, S , of a cone is the sum of its lateral area and the area of its base, that is, $S = L + B$. For a right cone, if the radius of the base is r and the slant height of the cone is ℓ , the surface area, S , is $S = \pi r \ell + \pi r^2$.

◆ Example 1

Find the exact surface area of the right cone and the surface area rounded to the nearest tenth.



◆ Solution

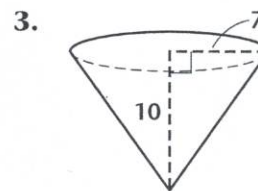
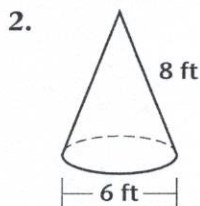
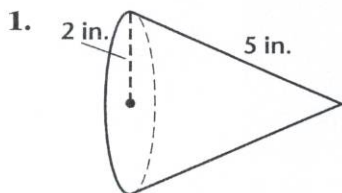
First, use the Pythagorean Theorem to find the slant height of the cone.

$$\ell^2 = 5^2 + 8^2 = 89; \ell = \sqrt{89}$$

$$S = \pi(5)(\sqrt{89}) + \pi(5^2) = 5\pi\sqrt{89} + 25\pi$$

The surface area of the cone is $5\pi\sqrt{89} + 25\pi$ square meters, or approximately 226.7 square meters.

Find the exact surface area of each right cone.



Find the exact surface area of each right cone. Then find the surface area to the nearest tenth.

4. radius of the base = 5 in., slant height = 12 in. _____
5. radius of the base = 2 mm, slant height = 6 mm _____
6. radius of the base = 7 ft, height = 8 ft _____
7. diameter of the base = 22 mm, height = 16 mm _____

◆ **Skill B** Finding the volume of a cone

Recall If the area of the base of a cone is B and the slant height of the cone is ℓ , then the volume, V , of the cone is $V = \frac{1}{3}Bh$. If the radius of the cone is r , then the volume is $V = \frac{1}{3}\pi r^2 h$.

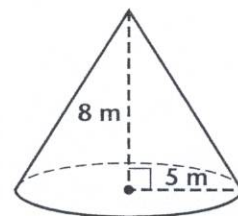
◆ **Example**

Find the exact volume of the cone. Then find the volume to the nearest hundredth.

◆ **Solution**

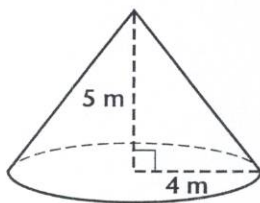
$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(5^2)(8) = \frac{200\pi}{3}$$

The volume of the cone is $\frac{200\pi}{3}$ cubic meters, or approximately 209.44 cubic meters.

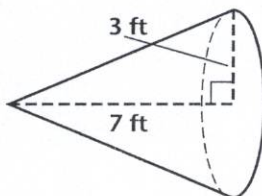


Find the exact volume of each cone.

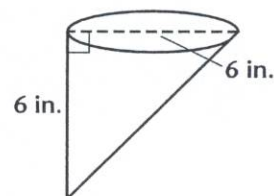
8.



9.



10.



Find the exact volume of each cone. Then find the volume rounded to the nearest tenth.

11. radius of the base = 18 in., height = 6 in. _____

12. radius of the base = 11 ft, height = 4 ft _____

13. radius of the base = 2.5 m, height = 4 m _____

Find the exact unknown value for each cone. In Exercises 16 and 17, the cones are right cones.

14. volume = 432π ft², height = 16 ft, radius of the base = _____

15. volume = 784π m², diameter of the base = 28 ft, height = _____

16. diameter of the base = 16 in., slant height = 17 in., volume = _____

17. radius of the base 9 mm, slant height = 41 mm, volume = _____

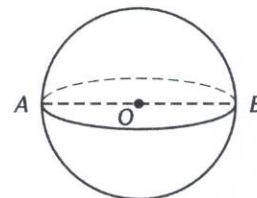


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7.6 Surface Area and Volume of Spheres

◆ Skill A Finding the surface area of a sphere

Recall In the figure at the right, O is the center of the sphere. \overline{OA} and \overline{OB} are radii and \overline{AB} is a diameter. The radius of the sphere is OB . The diameter is AB . The surface area, S , of a sphere with radius r is $S = 4\pi r^2$.



◆ Example

A sphere has radius 10 meters. Find the exact surface area of the sphere and the surface area rounded to the nearest tenth.

◆ Solution

Since $r = 10$, $S = 4\pi(10)^2 = 400\pi$.

The surface area of the sphere is 400π square meters, or approximately 1256.6 square meters.

Find the exact surface area of each sphere and the surface area rounded to the nearest hundredth.

1. radius = 3 in.

2. radius = 12 mm

3. radius = 6.2 cm

4. diameter = 16 yd

5. diameter = 7 ft

6. diameter = 4.6 in.

◆ Skill B Finding the volume of a sphere

Recall The volume, V , of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.

◆ Example

Find the exact volume of the sphere in the example above. Then find the volume to the nearest tenth of a cubic meter.

◆ Solution

$$r = 10, \text{ so } V = \frac{4}{3}\pi(10^3) = \frac{4000\pi}{3}.$$

The volume is $\frac{4000\pi}{3}$ cubic meters, or approximately 4188.8 cubic meters.

Find the exact volume of each sphere and the volume rounded to the nearest tenth.

7. radius = 24 yd

8. radius = 15 mm

9. radius = 11 m

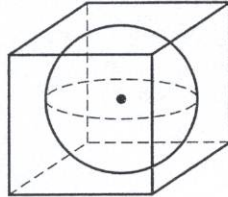
10. diameter = 18 in.

11. diameter = 17 yd

12. diameter = 13 cm

◆ Skill C Solving problems using the formula for the volume of a sphere**◆ Example**

The sphere is inscribed in the cube. The volume of the cube is 4096 cubic feet. Find the volume of the sphere.

**◆ Solution**

$$\text{volume of the cube} = 4096 = s^3; s = \sqrt[3]{4096} = 16$$

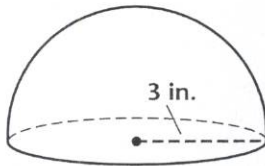
$$\text{radius of the sphere} = \frac{1}{2}s = 8$$

$$\text{volume of the sphere} = \frac{4}{3}\pi(8)^3$$

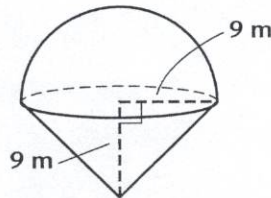
The volume of the sphere is $\frac{2048\pi}{3}$ cubic feet, or approximately 2144.66 square feet.

Find the exact volume of each space figure.

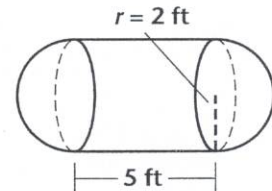
13.



14.



15.



16. A spherical scoop of ice cream has radius 1.25 inches. The scoop is placed on top of a cone with radius 1 inch and height 5 inches. Is the cone large enough to hold all the ice cream if it melts? Explain.
