Adding and Subtracting Polynomials

Learning Objectives:

1. Review combining like terms.
2. Know the vocabulary for polynomials.
3. Evaluate polynomials.
4. Add polynomials.
5. Subtract polynomials.
6. Add and subtract polynomials with more than one term.

Examples:

1. State the degree of the polynomial and whether it is a monomial, binomial, or trinomial.

   a) $2xy^7$ b) $12x^5 - 3x^2 + 2$ c) $9x^3y + 10x^4y^4$

2. Add.

   a) $(2x - 12) + (-3x + 22)$ b) $(5x^2 - 3x - 6) + (2x^2 - 8x - 3)$

   c) $\left(\frac{3}{4}x^2 - \frac{1}{3}x - 12\right) + \left(-\frac{1}{3}x^2 + \frac{2}{9}x + 3\right)$ d) $(-2.2x^3 + 5.4x - 0.1) + (6.4x - 3.4)$

3. Subtract.

   a) $(4x - 8) - (3x - 4)$ b) $(6x^3 + 5x^2 - 3) - (-2x^3 + 3x^2 - x + 1)$

   c) $\left(\frac{5}{8}x^2 - \frac{1}{3}x - 7\right) - \left(\frac{2}{3}x^2 - \frac{3}{4}x + 5\right)$ d) $(2.3x^4 - 4x^3 + 5x) - (x^4 - 6.2x^2 + 2.2x)$

4. Mixed practice.

   a) $(x^3 + 8x^2 + 5) + (6x - 3) - (3x^2 - x - 9)$

   b) $(7x^3 + 4x^2y + 7xy^2 - 3y^3) + (2x^3 + 5x^2y + 8xy^2) - (5x^2y + 2xy^2 + y^3)$

5. If it costs \$18 plus \$3 per day to rent a lawn dethatcher, the binomial $3x + 18$ gives the cost, in dollars, to rent the dethatcher for x days. How much will it cost to rent the dethatcher for 4 days?

Teaching Notes:

- Most students find this section easy.
- Remind students that this section is a review of distributing and collecting like terms.
- Some students like to set up the problems vertically aligning the like terms.
- Some students forget to distribute the minus sign when aligning vertically.

Answers: *1a) degree 8, monomial, b) degree 5, trinomial, c) degree 8, binomial; 2a)* $-x+10$, *b)* $7x^2-11x-9$, *c)* $(5/12)x^2-(1/9)x-9$, *d)* $-2.2x^3+11.8x-3.5$; *3a)* $x-4$, *b)* $8x^3+2x^2+x-4$, *c)* $(-1/24)x^2+(5/12)x-12$, *d)* $1.3x^4-4x^3+6.2x^2+2.8x$; *4a)* $x^3+5x^2+7x+11$, *b)* $9x^3+4x^2y+13xy^2-4y^3$; *5) \$30*

The Product Rule and Power Rules for Exponents

Learning Objectives:

1. Use exponents.
2. Use the product rule for exponents.
3. Use the rule $\left(a^m\right)^n = a^{mn}$.
4. Use the rule $(ab)^m = a^m b^m$.
5. Use the rule $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$.
6. Use combinations of the rules for exponents.
7. Use the rules for exponents in a geometry application

Examples:

1. Write in simplest exponent form.

   a) $3 \cdot 3 \cdot 3 \cdot x \cdot x$ b) $(-4)(m)(m)(n)(m)(n)$ c) $(5)(5)(x)(y)(x)(z)(y)(z)$

2. Multiply. Leave your answer in exponent form.

   a) $4^2 \cdot 4^3$ b) $x^8 \cdot x^6$ c) $(2p^4)(6p^5)$ d) $(-3y)(5y^7)(2y^3)$

   e) $(7x^4y^3)(3xy^6)$ f) $\left(\dfrac{2}{3}m^4n^3\right)\left(-\dfrac{3}{4}mn^6\right)$ g) $(2.3abc)(8.8a^2b^5)$

3. Divide. Leave your answer in exponent form.

   a) $\dfrac{6^5}{6^3}$ b) $\dfrac{x^7}{x^4}$ c) $\dfrac{y^3}{y^8}$ d) $\dfrac{6x^9}{2x^8}$

   e) $\dfrac{-12x^8y^3}{6x^{10}y}$ f) $\dfrac{24a^4b}{36a^4b^3}$ g) $\dfrac{-30x^0yz^3}{15yz^2}$ h) $\dfrac{4^2x^9y^5z^2}{4^0x^7y^4z^8}$

4. Raise exponential expressions to a power.

   a) $(x^2)^3$ b) $(a^2b^3)^4$ c) $(2xy^3)^5$ d) $\left(\dfrac{-6x}{y^3}\right)^3$

5. Mixed practice.

   a) $\dfrac{(8m^3)^4}{(8m)^2}$ b) $(4xy^3)^2(xy)$ c) $\left(\dfrac{a^2b^3}{c^5d}\right)^4$ d) $(-3a^2b^3)^2(ab^3)$

Teaching Notes:

- Students need a lot of repetition and practice in order to master these objectives.

Answers: 1a) 3^3x^2, b) $-4m^3n^2$, c) $5^2x^2y^2z^2$; 2a) 4^5, b) x^{14}, c) $12p^9$, d) $-30y^{11}$, e) $21x^5y^9$, f) $(-1/2)m^5n^9$, g) $20.24a^3b^6c$; 3a) 6^2, b) x^3, c) $1/y^5$, d) $3x$, e) $-2y^2/x^2$, f) $2/(3b^2)$, g) $-2z$, h) $16x^2y/z^6$; 4a) x^6, b) a^8b^{12}, c) $32x^5y^{15}$, d) $-216x^3/y^9$; 5a) $64m^{10}$, b) $16x^3y^7$, c) $a^8b^{12}/(c^{20}d^4)$, d) $9a^5b^9$

Multiplying Polynomials

Learning Objectives:

1. Multiply a monomial and a polynomial.
2. Multiply two polynomials.
3. Multiply binomials by the FOIL method.

Examples:

1. Multiply.

   a) $2x^2(6x-3)$ b) $4x(-2x^4+6x)$ c) $5x^3(-2x^3+7x-1)$

   d) $\frac{2}{3}(5x+6x^2-7x^3)$ e) $(3x^3+x^2-5x)(3xy)$ f) $(2x^2y^2-4xy+8)(-3xy)$

2. Multiply. Try to do most of the exercises mentally without writing down steps.

   a) $(x+2)(x+3)$ b) $(x+2)(x-6)$ c) $(x-4)(x-5)$

   d) $(3x-2)(-5x-6)$ e) $(4x-3y)(3x-4y)$ f) $(6x-3)^2$

   g) $(4y-3z)(5y-7z)$ h) $(0.5x+2)(6x-0.2)$ i) $\left(\frac{1}{2}x-\frac{1}{3}\right)\left(\frac{1}{3}x+\frac{1}{4}\right)$

3. Find the area of the rectangle with sides $(3x-4)$ and $(5x+3)$.

4. Multiply.

   a) $(x^3+2x^2-3x+1)(x+2)$ b) $(2x-3)(2x^3-3x^2+2x-1)$

5. What is wrong with each of the following multiplication answers?

   a) $(x-4)(-3)=3x-12$ b) $(3x+2)^2=9x^2+4$

Teaching Notes:

- Most students find this section easy.
- Remind students of the FOIL method, and encourage them to do it mentally.
- Some students like to multiply binomials in a different order than FOIL. Encourage them to use whatever order makes sense to them, as long as each term in one binomial distributes onto each term in the other binomial.
- Some students do not realize that $xy = yx$, and that they are therefore like terms.
- Many students distribute the exponent instead of multiplying the binomial by itself in problems such as 2f) and 5b).

Answers: 1a) $12x^3-6x^2$, b) $-8x^5+24x^2$, c) $-10x^6+35x^4-5x^3$, d) $(-14/3)x^3+4x^2+(10/3)x$, e) $9x^4y+3x^3y-15x^2y$, f) $-6x^3y^3+12x^2y^2-24xy$; 2a) x^2+5x+6, b) $x^2-4x-12$, c) $x^2-9x+20$, d) $-15x^2-8x+12$, e) $12x^2-25xy+12y^2$, f) $36x^2-36x+4$, g) $20y^2-43yz+21z^2$, h) $3x^2+11.9x-0.4$, i) $(1/6)x^2+(1/72)x-(1/12)$; 3) $15x^2-11x-12$; 4a) $x^4+4x^3+x^2-5x+2$, b) $4x^4-12x^3+13x^2-8x+3$ 5a) wrong signs: $-3x+12$, b) middle term is missing: $9x^2+12x+4$

Special Products

Learning Objectives:

1. Square binomials.
2. Find the product of the sum and difference of two terms.
3. Find greater powers of binomials.

Examples:

1. Use the formula $(a+b)(a-b)=a^2-b^2$ to multiply.

 a) $(x+2)(x-2)$ b) $(x+7)(x-7)$ c) $(3x-5)(3x+5)$

 d) $(4x-3)(4x+3)$ e) $(6a-b)(6a+b)$ f) $(3x-0.2)(3x+0.2)$

2. Use the formula for a binomial squared to multiply.

 a) $(3x-1)^2$ b) $(4x+5)^2$ c) $(7x-3)^2$

 d) $(3x+2y)^2$ e) $\left(\frac{2}{3}x+\frac{1}{4}\right)^2$ f) $(3y-4xz)^2$

3. Multiply.

 a) $x(2x+5)(2x-5)$ b) $(7p^2+1)(7p^2-1)$

 c) $(k+6)^3$ d) $(4m-3)^4$

Teaching Notes:

- Encourage students to memorize the formulas for multiplying a sum and a difference binomial and also a binomial squared. These will come in handy for factoring later.
- Some students have trouble remembering the formulas. Encourage them to think of multiplying polynomials as distributing. As long as each term in one polynomial distributes onto each term in the other polynomial, they will get the right answer.
- Some students have trouble keeping track of all the terms when multiplying the problems in number 3. Encourage them to start lining up like terms vertically as they do out the multiplications.
- Refer students to the ***Multiplying Binomials: A Sum and a Difference*** and ***A Binomial Squared*** charts in the textbook.

Answers: *1a) x^2-4, b) x^2-49, c) $9x^2-25$, d) $16x^2-9$, e) $36a^2-b^2$, f) $9x^2-0.04$; 2a) $9x^2-6x+1$, b) $16x^2+40x+25$, c) $49x^2-42x+9$, d) $9x^2+12xy+4y^2$, e) $(4/9)x^2+(1/3)x+(1/16)$, f) $9y^2-24xyz+16x^2z^2$; 3a) $4x^3-25x$, b) $49p^4-1$, c) $k^3+18k^2+108k+216$, d) $256m^4-768m^3+864m^2-432m+81$*

Dividing a Polynomial by a Monomial

Learning Objectives:

1. Divide a polynomial by a monomial.

Examples:

1. In the statement $\frac{5x^2-10}{5}$, the dividend is _____ and the divisor is _____ .

2. The expression $\frac{-7x+21}{7x}$ is undefined if _____ .

3. The problem $\frac{6x^3-3x^2+12x}{3x}=2x^2-x+4$ can be checked by multiplying _____ and _____ to show that the product is _____.

4. Divide.

   a) $\frac{20x^4-25x^2+20x}{5x}$ b) $\frac{27b^5-15b^3-9b^2}{3b^2}$ c) $(64x^7-32x^5+16x^3)\div 8x^3$

Teaching Notes:

- Some students need to see an arithmetic example of $\frac{a+b}{c}=\frac{a}{c}+\frac{b}{c}$ before trying 4.
- Encourage students to write the intermediate step shown in the above teaching note when dividing by a monomial. Otherwise, some students cancel the denominator with the first term in the numerator and do not know what to do next.
- Refer students to the ***Dividing a Polynomial by a Monomial*** in the textbook.

Answers: *1) dividend is $5x^2-10$ and 5 is the divisor; 2) $x=0$; 3) $2x^2-x+4$ and $3x$ to get $6x^3-3x^2+12x$; 4a) $4x^3-5x+4$, b) $9b^3-5b-3$, c) $8x^4-4x^2+2$*

Factors; The Greatest Common Factor

Learning Objectives:

1. Find the greatest common factor of a list of numbers.
2. Find the greatest common factor of a list of variable terms.
3. Factor out the greatest common factor.
4. Factor by grouping.

Examples:

1. Fill in the missing blank with the word "factors" or the word "terms".

   a) In the expression $5 \cdot 3$, the 5 and the 3 are called ________.
   b) In the expression 5 + 3, the 5 and the 3 are called ________.
   c) In the expression $5x^2 \cdot 3x^3$, the $5x^2$ and the $3x^3$ are called ________.
   d) In the expression $5x^2 + 3x^3$, the $5x^2$ and the $3x^3$ are called ________.

2. Find the greatest common factor for each list of numbers.

   a) 15, 27 b) 30, 18, 24

3. Remove the largest possible common factor. Check your answers by multiplication.

   a) $3x^2 + 6x$ b) $4y^2 + 4y$ c) $8a^2b^2 - 32ab$
   d) $15xy - 18yz - 27xz$ e) $36a^2 - 24ab - 16a$ f) $15x^2y - 25xy^2 + 20xy$

4. Factor completely.

   a) $3(2x + y) - z(2x + y)$ b) $7x(x - 5) + 3(x - 5)$ c) $4x(2y + 3z) - 7t(2y + 3z)$
   d) $12x^3 + 8x^2 - 15x - 10$ e) $(x + 2d) + 5c^2(x + 2d)$ f) $x(5y - 2) - (5y - 2)$
   g) $pq + 3q + 5p + 15$ h) $2b(y^2 - x) - 3a(y^2 - x) + 6c(y^2 - x)$

Teaching Notes:

- Some students need to rewrite the coefficients in problem 2 in factored form in order to identify the common factors.
- Encourage students to factor monomials in a step-by-step manner: first factor out the common number, then the common variable for each variable that exists.
- Many students omit the 1 in the answer for 3b). Encourage students to check their answers by multiplying to avoid this problem.
- Most students find it difficult to factor common binomials. Remind them that they are still factoring a common factor, as in problem 2, but the factor happens to be a binomial.

Answers: *1a) factors, b) terms, c) factors, d) terms; 2a) 3, b) 6; 3a) $3x(x+2)$, b) $4y(y+1)$, c) $8ab(ab-4)$, d) $3(5xy-6yz-9xz)$, e) $4a(9a-6b-4)$, f) $5xy(3x-5y+4)$; 4a) $(2x+y)(3-z)$, b) $(x-5)(7x+3)$, c) $(2y+3z)(4x-7t)$, d) $(3x+2)(4x^2-5)$, e) $(x+2d)(1+5c^2)$, f) $(5y-2)(x-1)$, g) $(p+3)(q+5)$, h) $(y^2-x)(2b-3a+6c)$*

Factoring Trinomials

Learning Objectives:

1. Factor trinomials with a coefficient of 1 for the squared term.
2. Factor trinomials after factoring out the greatest common factor.

Examples:

1. Find the two numbers that satisfy the requirements.
   a) Their product is 8 and their sum is 6
   b) Their product is 12 and their sum is 7
   c) Their product is –24 and their sum is –5
   d) Their product is –18 and their sum is 3

2. Factor.

   a) $x^2 + 3x + 2$ b) $x^2 + 6x + 8$ c) $x^2 - 6x + 8$ d) $x^2 - 10x + 9$

   e) $x^2 + x - 2$ f) $x^2 + 7x - 8$ g) $x^2 - 2x - 8$ h) $x^2 - 3x - 10$

   i) $x^2 + 12x + 35$ j) $x^2 + 2x - 48$ k) $x^2 - 11x + 24$ l) $x^2 - 4x - 21$

3. Factor out the greatest common factor. Then factor the remaining polynomial.

   a) $3x^2 + 21x + 30$ b) $5x^2 + 20x + 15$ c) $4x^2 - 8x - 96$

   d) $6x^2 + 6x - 72$ e) $3x^2 - 30x + 48$ f) $5x^2 - 20x - 105$

Teaching Notes:

- Tell students that factoring trinomials of the form $x^2 + bx + c$ is one of the most common forms of factoring they will use.
- In problems 2 and 3 many students find it helpful to make a table listing all possible factor pairs for c in the first column and their sums in the second column.
- Many students find it difficult at first to factor trinomials where the last term is negative.
- Remind students that when the last term is positive, the factor pairs they pick must have the same sign, while if the last term is negative, the factor pairs must have opposite signs.
- Refer students to the ***Factoring Trinomials of the Form $x^2 + bx + c$*** chart in the textbook.

Answers: 1a) 4 and 2, b) 4 and 3, c) –8 and 3, d) 6 and –3; 2a) (x+2)(x+1), b) (x+4)(x+2), c) (x–4)(x–2), d) (x–9)(x–1), e) (x+2)(x–1), f) (x+8)(x–1), g) (x–4)(x+2), h) (x–5)(x+2), i)(x+7)(x+5), j) (x+8)(x–6), k) (x–8)(x–3), l) (x–7)(x+3); 3a) 3(x+5)(x+2), b) 5(x+3)(x+1), c) 4(x–6)(x+4), d) 6(x+4)(x–3), e) 3(x–8)(x–2), f) 5(x–7)(x+3)

Factoring Trinomials by Grouping

Learning Objectives:

1. Factor trinomials by grouping when coefficient of the squared term is not 1.

Examples:

1. The middle term of each trinomial has been rewritten. Now factor by grouping.

a) $x^2+7x+10$
$=x^2+5x+2x+10$

b) $b^2+5b-24$
$=b^2-3b+8b-24$

c) $6c^2-13c-5$
$=6c^2+2c-15c-5$

2. Factor each trinomial by grouping.

a) $2y^2-y-10$ b) $6m^2+35m-6$ c) $12-s-s^2$

3. Factor the following expression as described below.

$3x^2-x+9x-3$

a) Factor by grouping the first two terms together and the last two terms together.

b) Factor by grouping the first and third terms together and the second and fourth terms together.

c) Do both methods give the same final answer?

Teaching Notes:

- Many students need the quick review of factoring a common binomial in problem 1 before attempting the factoring by grouping.
- Most students have trouble with signs when a negative must be factored out of the second grouping, as in problem 1c). Tell them that if the two binomials that remain after the first round of factoring have opposite signs, it means they should have pulled a negative sign out of the second grouping.
- Many students are amazed that two different grouping approaches can lead to the same final answer. Encourage them to try different groupings so that they will start to see patterns of which grouping arrangements make the factoring easier.

Answers: 1a) (x+5)(x+2), b) (b–3)(b+8), c) (3c+1)(2c–5); 2a) (2y–5)(y+2), b) (m+6)(6m–1), c) (3-s)(4+s); 3a) (3x–1)(x+3), b) (3x–1)(x+3), c) yes

Factoring Trinomials Using FOIL

Learning Objectives:

1. Factor trinomials using FOIL.

Examples:

1. Factor by the trial–and–error method. Check your answers using FOIL.

a) $2x^2 + 7x + 3$ b) $3x^2 - 2x - 8$ c) $5x^2 - 17x + 6$

2. Factor by the grouping method. Check your answers using FOIL.

a) $2x^2 + 7x + 3$ b) $3x^2 - 2x - 8$ c) $5x^2 - 17x + 6$

d) $6x^2 + 19x - 20$ e) $9x^2 + 29x + 6$ f) $8x^2 + 18x + 9$

g) $10y^2 + 23y + 12$ h) $6z^2 + 5z - 6$ i) $15z^2 - 4z - 4$

3. Factor out the greatest common factor from each term. Then factor the remaining trinomial.

a) $18x^2 - 78x - 60$ b) $10x^2 - 35x - 20$ c) $8y^2 + 44y + 20$

d) $-4x^2 + 10x + 6$ e) $9x^3 - 6x^2 - 24x$ f) $-45x^3 + 96x^2 - 48x$

Teaching Notes:

- Some students remember factoring trinomials from high school and do well with the trial-and-error method.
- Most students appreciate seeing the grouping method because it provides a recipe that works every time (providing the trinomial is factorable).
- Encourage students to use whichever method they prefer.
- In problem 2 most students find it helpful to make a table listing all possible factor pairs for *ac* in the first column and their sums in the second column.
- Remind students that when the last term is positive, the factor pairs they pick must have the same sign, while if the last term is negative, the factor pairs must have opposite signs.
- Many students are confused at first by problem 3d). Remind them to factor out the negative sign as a common factor.

Answers: 1a) $(2x+1)(x+3)$, b) $(3x+4)(x-2)$, c) $(5x-2)(x-3)$; 2a) $(2x+1)(x+3)$, b) $(3x+4)(x-2)$, c) $(5x-2)(x-3)$, d) $(6x-5)(x+4)$, e) $(9x+2)(x+3)$, f) $(4x+3)(2x+3)$, g) $(5y+4)(2y+3)$, h) $(3z-2)(2z+3)$, i) $(5z+2)(3z-2)$; 3a) $6(3x+2)(x-5)$, b) $5(2x+1)(x-4)$, c) $4(2y+1)(y+5)$, d) $-2(2x+1)(x-3)$, e) $3x(3x+4)(x-2)$, f) $-3x(5x-4)(3x-4)$

Special Factoring Techniques

Learning Objectives:

1. Factor a difference of squares.
2. Factor a perfect square trinomial.

Examples:

1. Factor by using the difference–of–two–squares formula.

   a) $x^2 - 4$ b) $x^2 - 49$ c) $9x^2 - 25$ d) $25x^2 - 49y^2$

   e) $100 - x^2$ f) $81x^4 - 1$ g) $81x^2 - y^2$ h) $x^4 - 4$

2. Factor by using the perfect–square trinomial formula.

   a) $x^2 + 18x + 81$ b) $x^2 + 2x + 1$ c) $x^2 + 12x + 36$

   d) $4x^2 - 28x + 49$ e) $49x^2 - 42xy + 9y^2$ f) $16x^2 + 72xy + 81y^2$

3. Factor by first looking for a greatest common factor.

   a) $4x^2 - 16$ b) $72x^2 - 98y^2$ c) $ab^2 - 16a$

   d) $18x^2 + 12x + 2$ e) $75x^2 + 90x + 27$ f) $125x^2 - 150x + 45$

4. Mixed review of factoring.

   a) $2x^2 + 6x + 4$ b) $3x^2 - 2x - 8$ c) $25x^2 - 100$

   d) $10x^2 - 35x - 20$ e) $9x^2 + 24xy + 16y^2$ f) $-x^2 - 2x + 48$

Teaching Notes:

- Some students understand the difference of squares formula better if 1a) and 1b) are first done using trinomial factoring (with a $0x$ middle term).
- Point out to students the importance of the phrase "perfect-square" trinomial. Encourage them to always check if the first and last terms of a trinomial are perfect squares.
- Encourage students to become proficient with special case factoring as it will be important for future algebra topics such as completing the square.

Answers: *1a)* $(x+2)(x-2)$, *b)* $(x+7)(x-7)$, *c)* $(3x+5)(3x-5)$, *d)* $(5x+7y)(5x-7y)$, *e)* $(10+x)(10-x)$, *f)* $(9x^2+1)(3x+1)(3x-1)$, *g)* $(9x+y)(9x-y)$, *h)* $(x^2+2)(x^2-2)$; *2a)* $(x+9)^2$, *b)* $(x+1)^2$, *c)* $(x+6)^2$, *d)* $(2x-7)^2$, *e)* $(7x-3y)^2$, *f)* $(4x+9y)^2$; *3a)* $4(x+2)(x-2)$, *b)* $2(6x+7y)(6x-7y)$, *c)* $a(b+4)(b-4)$, *d)* $2(3x+1)^2$, *e)* $3(5x+3)^2$, *f)* $5(5x-3)^2$; *4a)* $2(x+2)(x+1)$, *b)* $(3x+4)(x-2)$, *c)* $25(x+2)(x-2)$, *d)* $5(2x+1)(x-4)$, *e)* $(3x+4y)^2$, *f)* $-(x+8)(x-6)$

Solving Quadratic Equations by Factoring

Learning Objectives:

1. Solve quadratic equations by factoring.
2. Solve other equations by factoring.

Examples:

1. Solve for x using the zero factor property.

   a) $3 \cdot x = 0$ b) $2(x-5) = 0$ c) $x(x+6) = 0$ d) $(2x-3)(4x+5) = 0$

2. Using the factoring method, solve for the roots of each quadratic equation. Be sure to put the equation in standard form before factoring. Check your answers.

   a) $x^2 + 9x - 36 = 0$ b) $9x^2 - 2x = 0$

   c) $3x^2 - 21x + 30 = 0$ d) $5x^2 - 3x - 8 = 0$

   e) $x^2 - x = 6$ f) $x^2 - 64 = 63x$

   g) $x(3x+13) = 10$ h) $(x-3)(x+4) = -4(x+4)$

   i) $2 + \dfrac{x^2}{3} = 5x + 2$ j) $\dfrac{x^2 - 7x}{2} = 9$

Teaching Notes:

- Remind students to always put the equation in standard form before factoring.
- Some students try to use the zero factor property before the equation is in standard form. For example, 2e) $x^2 - x = 6 \rightarrow x(x-1) = 6 \rightarrow x = 6,\ x-1 = 6$ etc.

Answers: *1a) {0}, b) {5}, c) {–6, 0}, d) {–5/4, 3/2}; 2a) {–12, 3}, b) {0, 2/9}, c) {2, 5}, d) {–1, 8/5}, e) {–2, 3}, f) {–1, 64}, g) {–5, 2/3}, h) {–4, –1}, i) {0, 15}, j) {–2, 9}*

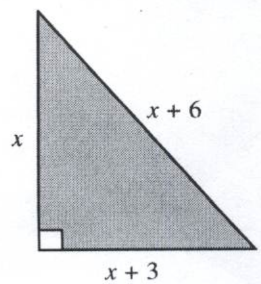
Applications of Quadratic Equations

Learning Objectives:

1. Solve problems about geometric figures.
2. Solve problems about consecutive integers.
3. Solve problems using the Pythagorean formula.
4. Solve problems using given quadratic models.

Examples:

1. The area of a circle is 144π square meters. Find its radius.
2. The width of a rectangle is 6 kilometers less than twice its length. If its area is 56 square kilometers, find the dimensions of the rectangle.
3. The product of two consecutive even integers is 14 more than 7 times their sum. Find the integers.
4. Find the lengths of the sides of the triangle.



5. A window washer accidentally drops a bucket from the top of a 144-foot building. The height, h, of the bucket after t seconds is given by $h = -16t^2 + 144$. When will the bucket hit the ground?

Teaching Notes:

- Many students find these problems difficult.
- Refer students to the ***Six-Step Process For Solving Application Problems*** in the text.
- Encourage students to make a diagram whenever possible.
- Many students find the applied problems difficult and need to see more examples.
- Remind students to check whether their answers are reasonable for applied problems.

<u>Answers</u>: 1) radius = 12 meters; 2) length = 8 km and width = 7 km; 3) 14 and 16; 4) x = 9, x+3 = 12, and x+6 = 15; 5) after 3 seconds

Solving Quadratic Equations by the Quadratic Formula

Learning Objectives:

1. Identify the values of *a*, *b*, and *c* in a quadratic equation.
2. Use the quadratic formula to solve quadratic equations.
3. Solve quadratic equations with only one solution.
4. Solve quadratic equations with fractions.

Examples:

1. Solve using the quadratic formula. If there are no real roots, say so.

   a) $x^2+5x+6=0$ b) $x^2+4x-7=0$ c) $3x^2-9x=-2$

   d) $3x^2-4x+8=0$ e) $5x^2=-10x-3$ f) $x^2+\frac{4}{5}x=-\frac{1}{5}$

2. Use the quadratic formula to find the roots.

   a) $9x^2-6x+1=0$ b) $25x^2+20x+4=0$

3. Determine which quadratic equation has no real roots by evaluating the discriminants.

   a) $x^2+3x+20=0$ b) $x^2+3x+2=0$

4. Solve for the variable. Choose the method you feel will work best.

   a) $8x^2-15x-2=0$ b) $(2x+9)^2=36$ c) $3x^2+10x=-6$

   d) $4x^2+6x+1=0$ e) $x^2+4x=0$ f) $(x+4)(x-3)=10$

   g) $x^2+x+3=0$ h) $x^2=-16x-100$ i) $3+x(x+4)=8$

Teaching Notes:

- Remind students to put the equation in standard form before identifying a, b, and c.
- Many students reduce final answers incorrectly: $\frac{4\pm\sqrt{5}}{8}\rightarrow\frac{1\pm\sqrt{5}}{2}$
- Some students prefer to always use the quadratic formula because it has no restrictions on when it can be used. Encourage them to also master the other methods, which are often quicker and easier to apply.

<u>Answers:</u> *1a)* $\{-3,-2\}$, *b)* $\{-2-\sqrt{11},-2+\sqrt{11}\}$, *c)* $\left\{\frac{9-\sqrt{57}}{6},\frac{9+\sqrt{57}}{6}\right\}$, *d)* $\varnothing$, *e)* $\left\{\frac{-5-\sqrt{10}}{5},\frac{-5+\sqrt{10}}{5}\right\}$, *f)* $\varnothing$; *2a)* $\left\{\frac{1}{3}\right\}$, *b)* $\left\{-\frac{2}{5}\right\}$; *3a)* $\varnothing$, *b) 2 real roots;* *4a)* $\left\{-\frac{1}{8},2\right\}$, *b)* $\left\{-\frac{15}{2},-\frac{3}{2}\right\}$, *c)* $\left\{\frac{-5-\sqrt{7}}{3},\frac{-5+\sqrt{7}}{3}\right\}$, *d)* $\left\{\frac{-3-\sqrt{5}}{4},\frac{-3+\sqrt{5}}{4}\right\}$, *e)* $\{-4,0\}$, *f)* $\left\{\frac{-1-\sqrt{89}}{2},\frac{-1+\sqrt{89}}{2}\right\}$, *g)* $\varnothing$, *h)* $\varnothing$, *i)* $\{-5,1\}$

Graphing Quadratic Equations

Learning Objectives:

1. Graph quadratic equations.
2. Find the vertex of a parabola.

Examples:

1. Sketch the parabolas $y = x^2$ and $y = -x^2$. Discuss the following points.

   a) The vertex is the lowest point (if parabola opens up) or highest point (if parabola opens down). Label the vertex on each parabola.

   b) Imagine shifting the parabolas around. How many x-intercepts can a parabola have?

   c) How many y-intercepts can a parabola have?

2. Find five ordered pairs for the equation. Then graph it.

   a) $y = x^2 + 1$ b) $y = x^2 - 2$ c) $y = \frac{1}{2}x^2$ d) $y = -\frac{1}{4}x^2$

   e) $y = -2x^2 + 2$ f) $y = (x+3)^2$ g) $y = (x-2)^2$ h) $y = \frac{1}{4}(x+1)^2$

3. Determine the vertex and the x-intercepts. Then sketch the graph.

   a) $y = x^2 - 1$ b) $y = x^2 + 4x$ c) $y = x^2 + 2x - 3$

   d) $y = -x^2 + 2x + 3$ e) $y = x^2 + 5x + 4$

Teaching Notes:

- Many students need to be told at first what x values to use when finding ordered pairs.
- Remind students that if their ordered pairs do not make the approximate vertex position apparent when graphing, then more ordered pairs are needed.
- Because graphing by ordered pairs is very time consuming for most students, it might work best to divide the equations in problem 2 among different groups of students.
- Most students are comfortable using the vertex formula, but some are confused at first by why the calculated value must be put back into the quadratic equation.
- Encourage students to also graph the y-intercepts in problem 3.

Answers: *1a) see graph answer pages, b) 0, 1, 2, c) 1; 2) see graph answer pages; 3a) vertex (0,–1), x–int (1,0) (–1,0), b) vertex (–2,–4), x–int (–4,0) (0,0), c) vertex (–1,–4), x–int (–3,0) (1,0), d) vertex (1,4), x–int(–1,0) (3,0), e) vertex (–5/2,–9/4), x–int (–4,0) (–1,0), see graph answer pages*